

Fully General Relativistic Simulations of Core-Collapse Supernovae with An Approximate Neutrino Transport

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ABSTRACT

We present results from the first generation of multi-dimensional hydrodynamic core-collapse simulations in full general relativity (GR) that include an approximate treatment of neutrino transport. Using a M1 closure scheme with an analytic variable Eddington factor, we solve the energy-independent set of radiation energy and momentum based on the Thorne's momentum formalism. To simplify the source terms of the transport equations, a methodology of multiflavour neutrino leakage scheme is partly employed. Our newly developed code is designed to evolve the Einstein field equation together with the GR radiation hydrodynamic equations in a self-consistent manner while satisfying the Hamiltonian and momentum constraints. An adaptive-mesh-refinement technique implemented in the three-dimensional (3D) code enables us to follow the dynamics starting from the onset of gravitational core-collapse of a $15 M_{\odot}$ star, through bounce, up to about 100 ms postbounce in this study. By computing four models that differ according to 1D to 3D and by switching from special relativistic (SR) to GR hydrodynamics, we study how the spacial multi-dimensionality and GR would affect the dynamics in the early postbounce phase. Our 3D results support the anticipation in previous 1D results that the neutrino luminosity and average neutrino energy of any neutrino flavor in the postbounce phase increase when switching from SR to GR hydrodynamics. This is because the deeper gravitational well of GR produces more compact core structures, and thus hotter neutrino spheres at smaller radii. By analyzing the residency timescale to the neutrino-heating timescale in the gain region, we show that the criterion to initiate neutrino-driven explosions can be most easily satisfied in 3D models, irrespective of SR or GR hydrodynamics. Our results suggest that the combination of GR and 3D hydrodynamics provides the most favorable condition to drive a robust neutrino-driven explosion. To draw a solid conclusion, the energy and angle dependence of neutrino transport should be accurately incorporated in our full GR code with the use of more detailed set of weak interactions. This work is the very first step before going up the long and winding road.

Subject headings: supernovae: collapse — neutrinos — hydrodynamics—general relativity

1. Introduction

Core-collapse supernova simulations have been counted as one of the most challenging subjects in computational astrophysics. The four fundamental forces of nature are all at play; the collapsing iron core bounces due to strong interactions; weak interactions determine the energy and lepton number loss in the core via the transport of neutrinos; electromagnetic interactions determine the properties of the stellar gas; general relativity plays an important role due to the compactness of the proto-neutron star and also due to high velocities of the collapsing material outside. Naturally, such physical richness ranging from a microphysical scale (i.e. femto-meter scale) of strong/weak interactions to a macrophysical scale of stellar explosions has long attracted the interest of researchers, necessitating a world-wide, multi-disciplinary collaboration to clarify the theory of massive stellar core-collapse and the formation mechanisms of compact objects.

Ever since the first numerical simulation of such events (Colgate & White 1966), the neutrino-heating mechanism (Wilson 1985; Bethe & Wilson 1985), in which a stalled bounce shock could be revived via neutrino absorption on a timescale of several hundred milliseconds after bounce, has been the working hypothesis of supernova theorists for these ~ 45 years. However, the simplest, spherically-symmetric (1D) form of this mechanism fails to blow up canonical massive stars (Rampp & Janka 2000; Liebendörfer et al. 2001; Thompson et al. 2003; Sumiyoshi et al. 2005). Pushed by mounting supernova observations of the blast morphology (e.g., Wang et al. (2001); Maeda et al. (2008); Tanaka et al. (2009), and references therein), it is now almost certain that the breaking of the spherical symmetry holds the key to solve the supernova problem. So far a number of multidimensional (multi-D) hydrodynamic simulations have shown that hydrodynamic motions associated with convective overturn (e.g., Herant et al. (1992); Burrows et al. (1995); Janka & Müller (1996); Fryer et al. (2002); Fryer (2004)) and the Standing-Accretion-Shock-Instability (SASI, e.g., Blondin et al. (2003); Scheck et al. (2004); Scheck et al. (2006); Ohnishi et al. (2006); Ohnishi et al. (2007); Foglizzo et al. (2006); Iwakami et al. (2008); Iwakami et al. (2009); Murphy & Burrows (2008); Fernández & Thompson (2009b,a), and references therein) can help the onset of the neutrino-driven explosion.

In fact, the neutrino-driven explosions have been obtained in the following first-principle two-(2D) and three-(3D) dimensional simulations in which the spectral neutrino transport is solved at various levels of approximations (e.g., Müller et al. (2011); Ott et al. (2011); Kotake (2011) for recent status reports). Among them are the 2D neutrino-radiation-hydrodynamic simulations by Buras et al. (2006a,b); Marek & Janka (2009) who included one of the best available neutrino transfer approximations by the ray-by-ray variable Eddington factor method, by Bruenn et al. (2010) who included a ray-by-ray multi-group flux-limited diffusion (MGFLD) transport with the best available weak interactions, and by Suwa et al. (2010, 2011) who employed a ray-by-ray isotropic diffusion source approximation (IDSA) (Liebendörfer et al. 2009) with a reduced set of weak interactions. By extending the 2D modules in Suwa et al. (2010), Takiwaki et al. (2011) recently reported neutrino-driven explosion models in 3D for an $11.2 M_{\odot}$ star. They pointed out whether 3D effects would help explosions or not is sensitive to the employed numerical resolutions (see also Hanke et al.

(2011); Nordhaus et al. (2010)). They argued that future peta- and exa-scale resources are at least needed to draw a robust conclusion of the 3D effects.

In addition to the 3D effects, impacts of general relativity (GR) on the neutrino-driven mechanism stand out among the biggest open questions in the supernova theory. It should be remembered that using newly derived Einstein equations (Misner & Sharp 1964), the consideration of GR was standard in the pioneering era of supernova simulations (e.g., May & White (1966)). One year after Colgate & White (1966), Schwartz (1967) reported the first fully GR simulation of stellar collapse to study the supernova mechanism, who implemented a gray transport of neutrino diffusion in the 1D GR hydrodynamics¹. Using GR Boltzmann equations derived by Lindquist (1966), Wilson (1971) developed a 1D GR-radiation-hydrodynamic code including a more realistic (at the time) description of the collisional term than the one in Schwartz (1967). By performing 1D GR hydrodynamic simulations that included a leakage scheme for neutrino cooling, hydrodynamical properties up to the prompt shock stagnation were studied in detail (van Riper 1979; van Riper & Lattimer 1981; van Riper 1982). These pioneering studies, albeit using a much simplified neutrino physics than today, did provide a bottom-line of our current understanding of the supernova mechanism (see Bruenn et al. (2001) for a complete list of references for the early GR studies). In the middle of the 1980s, Bruenn (1985) developed a code that coupled 1D GR hydrodynamics to the MGFLD transport of order (v/c) including the so-called standard set of neutrino interactions. Since the late 1990s, the ultimate 1D simulations, in which the GR Boltzmann transport is coupled to 1D GR hydrodynamics, have been made feasible by Sumiyoshi-Yamada et al. (Yamada 1997; Yamada et al. 1999; Sumiyoshi et al. 2005, 2007)² and by Liebendörfer-Mezzacappa-Bruenn et al. (Mezzacappa & Matzner 1989; Bruenn et al. 2001; Liebendörfer et al. 2001; Liebendörfer et al. 2001, 2004) (and by their collaborators).

Among them, Bruenn et al. (2001) presented evidence that average neutrino energy of any neutrino flavor during the shock reheating phase increase when switching from Newtonian to GR hydrodynamics. They also pointed out that the increase is larger in magnitude compared to the decrease due to redshift effects and gravitational time dilation. By employing the currently best available weak interactions, Lentz et al. (2011) very recently reported the update of Bruenn et al. (2001). They showed that the omission of observer corrections in the transport equation particularly does harm to drive the neutrino-driven explosions. In these full-fledged 1D simulations, a commonly observed disadvantageous aspect of GR to drive neutrino-driven explosions is that the residency time of material in the gain region becomes shorter due to the stronger gravitational pull. As a result of these competing ingredients in the end, GR works disadvantageously to facilitate the

¹Citing from his paper, "*In this calculation, the neutrino luminosity of the core is found to be 10^{54} erg/s, or 1/2 a solar rest mass per second !! This is the mechanism which the supernova explodes*". The neutrino luminosity rarely becomes so high in the modern simulations, but it is surprising that the potential impact of GR on the neutrino-heating mechanism was already indicated in the very first GR simulation.

²Very recently, they reported their success to develop the first multi-angle, multi-energy neutrino transport code in 3D (Sumiyoshi & Yamada 2012).

neutrino-driven explosions in 1D. In fact, the maximum shock extent in the postbounce phase is shown to be 20% smaller when switching from Newtonian to GR hydrodynamics (e.g., Figure 2 in Lentz et al. (2011)).

Among the most up-to-date multi-D models with spectral neutrino transport mentioned earlier, GR effects are at best attempted to be modeled by using a modified gravitational potential that takes into account a 1D, post-Newtonian correction (Buras et al. 2006a,b; Marek & Janka 2009; Bruenn et al. 2010). A possible drawback of this prescription is that a conservation law for the total energy cannot be guaranteed by adding an artificial term to the Poisson equation of self-gravity. Since the energy reservoir of the supernova engines is the gravitational binding energy, any potential inaccuracies in the argument of gravity would be better eliminated. There are a number of relativistic simulations of massive stellar collapse in full GR (e.g., 2D (Shibata & Sekiguchi 2005a) or 3D (Shibata & Sekiguchi 2005b; Ott et al. 2007), and references therein) or using the conformally-flatness approximation (CFC) (e.g., Dimmelmeier et al. (2002); Cordero-Carrión et al. (2009)). Although extensive attempts have been made to include microphysics such as by the Y_e formula (Liebendörfer 2005) or by a neutrino leakage scheme (Sekiguchi 2010), the effects of neutrino heating have yet to be included in them, which is a main hindrance to study the GR effects on the multi-D neutrino-driven mechanism³.

In this paper, we present a new fully GR code for multi-D hydrodynamic supernova simulations in which an approximate neutrino transport is implemented. The code is a marriage of an adaptive-mesh-refinement (AMR), conservative 3D GR magnetohydrodynamic (MHD) code developed by Kuroda & Umeda (2010), and the approximate neutrino transport code that we newly develop in this work. The spacetime treatment in our full GR code is based on the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formalism (see, e.g., Shibata & Nakamura 1995; Baumgarte & Shapiro 1999). The hydrodynamics can be solved either in full GR or in special relativity (SR), which allows us to investigate the GR effects on the supernova dynamics. Using a M1 closure scheme with an analytic variable Eddington factor, we solve the energy-independent set of radiation energy and momentum. This part is based on the partial implementation of the Thorne’s momentum formalism, which is recently extended by Shibata et al. (2011) in a more suitable manner applicable to the neutrino transport problem. Similar to the IDSA scheme (Liebendörfer et al. 2009), we conceptually divide the neutrinos into two parts, which are ”trapped” and ”free-streaming” neutrinos. By doing so, we model the source terms of the transport equations to be expressed in a simplified manner with the use of a multi-flavor neutrino leakage scheme (e.g., Rosswog & Liebendörfer (2003)). Our newly developed code is designed to evolve the Einstein field equation together with the GR radiation hydrodynamic equations in a self-consistent manner while satisfying the Hamiltonian and momentum constraints. An adaptive-mesh-refinement technique implemented in the 3D code enables us to follow the dynamics starting from the onset of gravitational core-collapse of a 15

³Very recently, Müller et al. (2012) reported explosions for 11.2 and 15 M_\odot stars based on their 2D GR simulations in CFC with detailed neutrino transport (Müller et al. 2010) similar to Buras et al. (2006a).

M_\odot star, through bounce, up to about 100 ms postbounce in this study. For the $15 M_\odot$ star, the neutrino-driven explosions are expected to take place later than ~ 200 ms postbounce at the earliest (e.g., Bruenn et al. (2010); Marek & Janka (2009)). However it is computationally too expensive to follow such a long-term evolution in our full 3D GR simulations. Albeit limited to the rather early postbounce phase, we would exploratory study possible GR effects in the multi-D neutrino-driven mechanism by comparing 1D to 3D results and by switching from SR to GR hydrodynamics.

This paper is organized as follows: In section 2, after we introduce the model concept of the approximate GR transport scheme, we summarize the governing equations of hydrodynamics and neutrino transport in detail. The main results are presented in Section 3. We summarize our results and discuss their implications in Section 4. Note that geometrized units are used throughout Sections 2 to 3, i.e. both the speed of light and the gravitational constant are set to unity: $G = c = 1$. Greek indices run from 0 to 3, Latin indices from 1 to 3.

2. Basic Equations for General Relativistic Neutrino-Radiation Hydrodynamics

Our newly developed code consists of the three parts, in which the evolution equations of metric, hydrodynamics, and neutrino radiation are solved, respectively. As will be mentioned, each of them is solved in an operator-splitting manner, but the system evolves self-consistently as a whole satisfying the Hamiltonian and momentum constraints. Before going into details, we shortly describe the bottom-line how to add radiation to GR hydrodynamics.

The starting-point is the conservation of the total energy (fluid + radiation),

$$\nabla_\alpha T_{(\text{total})}^{\alpha\beta} = \nabla_\alpha T_{(\text{fluid})}^{\alpha\beta} + \nabla_\alpha T_{(\nu)}^{\alpha\beta} = 0, \quad (1)$$

where $T_{(\text{total})}^{\alpha\beta}$, $T_{(\text{fluid})}^{\alpha\beta}$, and $T_{(\nu)}^{\alpha\beta}$ is the stress-energy tensor of the total energy, fluid, and neutrino radiation, respectively. Then Equation (1) can be decomposed as,

$$\nabla_\alpha T_{(\text{fluid})}^{\alpha\beta} = -Q^\beta, \quad (2)$$

and

$$\nabla_\alpha T_{(\nu)}^{\alpha\beta} = Q^\beta, \quad (3)$$

where Q^β represents the source terms that describe the exchange of energy and momentum between fluid and radiation. If Q^β would be given, it is rather straightforward to evolve Equation (2) following standard procedures in numerical relativity. Accordingly, what we focus on in this section is how to determine the source terms Q^β and the evolution equations of neutrinos (i.e. the concrete form of the left-hand-side of Equation (3)). In doing so, there will appear many terms related to GR such as $e^{6\phi}$, $\partial_i \beta^j$.. etc. So, after we briefly summarize the BSSN formalism in the next section, we first present the transport equations in section 2.2 and then the source terms in section 2.3.

2.1. Metric equations

We write the spacetime metric in the standard (3+1) form:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (4)$$

where α , β^i , and γ_{ij} are the lapse, shift, and spatial metric, respectively. The extrinsic curvature K_{ij} is defined by

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}, \quad (5)$$

where \mathcal{L}_β is the Lie derivative with respect to β^i . The evolution of γ_{ij} and K_{ij} is governed by the Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}(\text{total})$, where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}(\text{total})$ is the total stress-energy tensor (e.g., Equation (1)).

We evolve γ_{ij} and K_{ij} using the BSSN formulation (Baumgarte & Shapiro 1999; Shibata & Sekiguchi 2005b; Duez et al. 2006), in which the fundamental variables are

$$\phi \equiv \frac{1}{12} \ln[\det(\gamma_{ij})], \quad (6)$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi} \gamma_{ij}, \quad (7)$$

$$K \equiv \gamma^{ij} K_{ij}, \quad (8)$$

$$\tilde{A}_{ij} \equiv e^{-4\phi} (K_{ij} - \frac{1}{3} \gamma_{ij} K), \quad (9)$$

$$\tilde{\Gamma}^i \equiv -\tilde{\gamma}^{ij}{}_{,j}. \quad (10)$$

The Einstein equation gives rise to the evolution equations for the BSSN variables as,

$$(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} \quad (11)$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\frac{1}{6}\alpha K \quad (12)$$

$$(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} = e^{-4\phi} \left[\alpha(R_{ij} - 8\pi\gamma_{i\mu}\gamma_{j\nu}T_{(\text{total})}^{\mu\nu} - D_i D_j \alpha) \right]^{\text{trf}} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{\gamma}^{kl}\tilde{A}_{jl}) \quad (13)$$

$$(\partial_t - \mathcal{L}_\beta)K = -\Delta\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + K^2/3) + 4\pi\alpha(n_\mu n_\nu T_{(\text{total})}^{\mu\nu} + \gamma^{ij}\gamma_{i\mu}\gamma_{j\nu}T_{(\text{total})}^{\mu\nu}) \quad (14)$$

$$\begin{aligned} (\partial_t - \beta^k \partial_k)\tilde{\Gamma}^i &= 16\pi\tilde{\gamma}^{ij}\gamma_{i\mu}n_\nu T_{(\text{total})}^{\mu\nu} \\ &\quad - 2\alpha\left(\frac{2}{3}\tilde{\gamma}^{ij}K_{,j} - 6\tilde{A}^{ij}\phi_{,j} - \tilde{\Gamma}_{jk}^i \tilde{A}^{jk}\right) \\ &\quad + \tilde{\gamma}^{jk}\beta_{,jk}^i + \frac{1}{3}\tilde{\gamma}^{ij}\beta_{,kj}^k - \tilde{\Gamma}^j\beta_{,j}^i + \frac{2}{3}\tilde{\Gamma}^i\beta_{,j}^j + \beta^j\tilde{\Gamma}_{,j}^i - 2\tilde{A}^{ij}\alpha_{,j}, \end{aligned} \quad (15)$$

where D denotes covariant derivative operator associated with γ_{ij} , $\Delta = D^i D_i$, “trf” denotes the trace-free operator, $n_\mu = (-\alpha, 0)$ is the time-like unit vector normal to the $t = \text{constant}$ time slices. In Equation (13), the explicit form of $D_i D_j \alpha$ reads

$$\begin{aligned} D_i D_j \alpha &= \partial_i \partial_j \alpha - \Gamma_{ij}^k \partial_k \alpha \\ &= \partial_i \partial_j \alpha - \left[\tilde{\Gamma}_{ij}^k + 2\left(\delta_j^k \partial_i \phi + \delta_i^k \partial_j \phi - \tilde{\gamma}_{ij} \tilde{\gamma}^{kl} \partial_l \phi \right) \right] \partial_k \alpha. \end{aligned} \quad (16)$$

Following Alcubierre & Brügmann (2001), the gauge is specified by the 1+log lapse,

$$\partial_t \alpha = \beta^i \partial_i \alpha - 2\alpha K, \quad (17)$$

and by the Gamma-driver-shift,

$$\partial_t \beta^i = k \partial_t \tilde{\Gamma}^i, \quad (18)$$

here we chose $k = 1$. For further information with code verification of the metric solver, see Kuroda & Umeda (2010)⁴. Having summarized the bottom-line of the BSSN formalism, we are now ready to discuss the transport equations.

2.2. Neutrino Transport Equations

To determine the transport equations in GR, we follow the truncated momentum formalism (Thorne 1981), which is recently extended by Shibata et al. (2011) in a suitable form for the neutrino transport problem. The starting point is to define the radiation stress-energy tensor as,

$$T_{(\nu)}^{\alpha\beta} \equiv E_{(\nu)} n^\alpha n^\beta + F_{(\nu)}^\alpha n^\beta + F_{(\nu)}^\beta n^\alpha + P_{(\nu)}^{\alpha\beta}, \quad (19)$$

where $E_{(\nu)}$, $F_{(\nu)}$, and $P_{(\nu)}$, is the radiation energy, flux, pressure measured in the laboratory frame, respectively. Conversely, $E_{(\nu)}$, $F_{(\nu)}^\alpha$, and $P_{(\nu)}^{\alpha\beta}$ are given by the stress-energy tensor as,

$$E_{(\nu)} = T_{(\nu)}^{\alpha\beta} n_\alpha n_\beta, \quad F_{(\nu)}^i = -T_{(\nu)}^{\alpha\beta} n_\alpha \gamma_\beta^i, \quad P_{(\nu)}^{ij} = T_{(\nu)}^{\alpha\beta} \gamma_\alpha^i \gamma_\beta^j. \quad (20)$$

In the following, radiation variables are all defined in the laboratory frame unless otherwise stated.

According to Shibata et al. (2011), the evolution equations of radiation energy and radiation flux in Equation (19) can be written as

$$\partial_t(e^{6\phi} E_{(\nu)}) + \partial_i[e^{6\phi}(\alpha F_{(\nu)}^i - \beta^i E_{(\nu)})] = e^{6\phi}(\alpha P^{ij} K_{ij} - F_{(\nu)}^i \partial_i \alpha - \alpha Q^\mu n_\mu), \quad (21)$$

and

$$\partial_t(e^{6\phi} F_{(\nu)_i}) + \partial_j[e^{6\phi}(\alpha P_{(\nu)_i}^j - \beta^j F_{(\nu)_i})] = e^{6\phi}[-E_{(\nu)} \partial_i \alpha + F_{(\nu)_j} \partial_i \beta^j + (\alpha/2) P_{(\nu)}^{jk} \partial_i \gamma_{jk} + \alpha Q^\mu \gamma_{i\mu}], \quad (22)$$

where Q^μ denotes the source terms. For the three radiation variables ($E_{(\nu)}$, $F_{(\nu)}^i$, $P_{(\nu)}^{ij}$) in Equations (21,22), we have only two sets of the equation. Here we employ the so-called M1 closure (Levermore 1984), in which the radiation pressure is related to the radiation energy and flux by an analytical closure relation (i.e. $P_{(\nu)}(E_{(\nu)}, F_{(\nu)})$) as,

$$P_{(\nu)}^{ij} = \frac{3\chi - 1}{2} P_{\text{thin}}^{ij} + \frac{3(1 - \chi)}{2} P_{\text{thick}}^{ij}, \quad (23)$$

⁴In Kuroda & Umeda (2010), an auxiliary variable $F_i \equiv \delta^{jk} \partial_k \tilde{\gamma}_{ij}$ was evolved instead of $\tilde{\Gamma}^i$.

where χ represents the variable Eddington factor, P_{thin}^{ij} and P_{thick}^{ij} corresponds to the radiation pressure in the optically thin and thick limit, respectively.

For the variable Eddington factor χ , we employ the one proposed by Levermore (1984),

$$\chi = \frac{3 + 4\bar{F}^2}{5 + 2\sqrt{4 - 3\bar{F}^2}}, \quad (24)$$

$$\bar{F}^2 \equiv \frac{F^i F_i}{E^2}. \quad (25)$$

It can be readily checked that in the optically thick limit, $\chi \rightarrow 1/3$ because $F^i \rightarrow 0$, then $P^{ij} \rightarrow P_{\text{thick}}^{ij}$, while in the optically thin limit, $\chi \rightarrow 1$ because $\bar{F}^2 \rightarrow 1$, then $P^{ij} \rightarrow P_{\text{thin}}^{ij}$.

Following Audit et al. (2002); Shibata et al. (2011), the following forms of P_{thin}^{ij} and P_{thick}^{ij} are adopted,

$$P_{\text{thin}}^{ij} = E \frac{F^i F^j}{F_k F^k}, \quad (26)$$

and

$$P_{\text{thick}}^{ij} = \mathcal{J} \frac{\gamma^{ij} + 4\gamma^{ik}\gamma^{jl}u_k u_l}{3} + \gamma^{jk}\mathcal{H}^i u_k + \gamma^{ik}\mathcal{H}^j u_k, \quad (27)$$

respectively. By this choice, the radiation flux naturally changes with radius (r) as $\sim 1/r^2$ in the low opacity regime (e.g., Appendix C). This may sound quite straightforward, but it is one of the most important issue for the purpose of this work, because the radiation neutrino flux in the semi-transparent regions holds the key to the success or failure of the neutrino-heating mechanism. \mathcal{J}, \mathcal{H} in Equation (27) denotes the Eddington moments in the comoving frame, which are related to those in the laboratory frame as,

$$\mathcal{J} = E_{(\nu)} W^2 - 2W F_{(\nu)}^i u_i + P_{(\nu)}^{ij} u_i u_j, \quad (28)$$

and

$$\mathcal{H}^\alpha = (E_{(\nu)} W - F_{(\nu)}^i u_i)(n^\alpha - W u^\alpha) + W h_\beta^\alpha F_{(\nu)}^\beta - h_i^\alpha u_j P_{(\nu)}^{ij}, \quad (29)$$

where $W = \alpha u^0$ is the Lorentz factor, $h_{\alpha\beta}$ is the projection operator defined by

$$h_{\alpha\beta} \equiv g_{\alpha\beta} + u_\alpha u_\beta. \quad (30)$$

Having summarized the closed set of the two-moment transport equations, we are now going to discuss the source terms ($;Q^\mu$) in the next section.

2.3. Source terms

To model the source terms, we follow the idea of the IDSA scheme (Liebendörfer et al. 2009), in which neutrinos are conceptually divided into two parts, which are "trapped" and "free-streaming"

neutrinos, respectively. We also utilize a methodology of multi-flavor neutrino leakage scheme (e.g., Rosswog & Liebendörfer (2003)) to simplify the description of the source terms.

Figure 1 illustrates the concept how to estimate the source terms. To describe the neutrino-matter coupling, we need to ask at least three actors, namely "matter", "trapped neutrino", and "streaming neutrino", to appear in the playground of the supernova core. The neutrino sphere is an important quantity to describe the relationship between them. The position of the neutrino sphere⁵ at which the neutrino optical depth comes close to unity, is indicated by $\tau_\nu = 2/3$ in Figure 1. The trapped neutrinos are always coupled with matter (through β -equilibrium) and their temperature is the same as that of matter. On the other hand, temperature of the free-streaming neutrinos cannot be determined locally owing to its decoupling nature from matter, which was the reason that we have to solve the evolution equations. So these two represent a two extreme limit.

In Figure 1, trapped neutrinos are denoted by " ν_{trap} " (in a diamond shape colored by grey), which is illustrated to shake hands with matter inside the neutrino sphere (inside the region enclosed by $\tau_\nu = 2/3$). There the trapped neutrinos dominate over the streaming neutrinos (denoted by " ν_{stream} " (in a jaggy circle colored by orange) in Figure 1), which is vice versa outside the neutrino sphere. This is illustrated in such a way that " ν_{trap} " is bigger than " ν_{stream} " inside the neutrino sphere, which is vice versa outside the neutrino sphere. For " ν_{stream} " outside the neutrino sphere, the jaggy circle is drawn to have several tails, by which we intend to express that it can travel much more freely in the free-streaming regime.

In Figure 1, the three actors are connected by thick arrows (in blue or red), each of them is labeled by $Q_{diff}^{\mu,C}$ (in blue), $Q_{intr}^{\mu,C}$ (in blue), or $Q^{\mu,H}$ (in red), representing the couplings in-between. The arrows colored by blue ($Q_{diff}^{\mu,C}$, $Q_{intr}^{\mu,C}$) represent neutrino cooling, while the arrow in red ($Q^{\mu,H}$) does neutrino heating. The neutrino cooling means that energy is transferred from matter (or from trapped neutrinos) to streaming neutrinos that carry the imparted energy away from the system. On the other hand, the neutrino heating proceeds by energy transfer from streaming neutrinos to matter (see $Q^{\mu,H}$ in Figure 1). Finally $Q_{diff}^{\mu,C}$ in Figure 1 represents the cooling by neutrinos leaking out from opaque regions inside the neutrino sphere by diffusion.⁶

Looking at Figure 1 again, the source term of Equations (21,22) can be readily defined as,

$$Q^\mu \equiv Q^{\mu,C} - Q^{\mu,H}, \quad (31)$$

where each of the cooling ($Q^{\mu,C}$) and heating($Q^{\mu,H}$) term is calculated in the present scheme as,

$$Q^{\mu,C} = \sum_{\nu \in \nu_e, \bar{\nu}_e, \nu_x} \left[(1 - e^{-\beta_\nu \tau_\nu}) Q_{\nu, diff} + e^{-\beta_\nu \tau_\nu} Q_{\nu, intr}^C \right] u^\mu, \quad (32)$$

⁵here defined for the average neutrino energy for simplicity,

⁶Note inside the neutrino sphere, neutrino heating locally balances with neutrino cooling by weak interactions due to β -equilibrium. So as a net, the diffusion-mediated cooling becomes dominant there.

$$Q^{\mu,H} = \sum_{\nu \in \nu_e, \bar{\nu}_e} e^{-\beta_\nu \tau_\nu} \varepsilon_\nu^2 \tilde{\kappa}_\nu (-\mathcal{J}_\nu u^\mu - \mathcal{H}_\nu^\mu). \quad (33)$$

Before we go into details, we need to draw a caution that we introduced the concept of the streaming and trapped neutrinos only for the sake of (better) explanation of our approximate treatment. Actually the sum of the trapped and streaming is transported by Equations (21,22) with the source terms described above. We design the source terms in such a way to connect the heating/cooling terms smoothly between the diffusion and free-streaming limit, which is basically similar to the concept of the M1 closure relation.

The cooling term ($Q^{\mu,C}$) consists of $Q_{\nu,diff}$ and $Q_{\nu,intr}^C$, which accounts for neutrino cooling by diffusion out of the neutrino sphere and the one determined locally outside the neutrino sphere, respectively (e.g., Figure 1). Following van Riper & Lattimer (1981), the terms of $1 - e^{-\beta_\nu \tau_\nu}$ and $e^{-\beta_\nu \tau_\nu}$ appearing in Equation (32) are introduced to smoothly connect the two quantities ($Q_{\nu,diff}$ and $Q_{\nu,intr}^C$) for the semi-transparent regime. Here τ_ν represents the optical depth of neutrinos and β_ν is a model parameter that we determine by the comparison with a spectral neutrino transport calculation (see Appendix A.1). With these terms bridging the two limits, it is easy to see that $Q^{\mu,C}$ approaches to $Q_{\nu,diff}$ for the diffusion limit ($\tau_\nu \rightarrow \infty$), and it does to $Q_{\nu,intr}^C$ for the free-streaming limit ($\tau_\nu \rightarrow 0$).

Following the neutrino leakage scheme (e.g., Epstein & Pethick (1981); van Riper & Lattimer (1981); Kotake et al. (2003)), the diffusion cooling rate ($Q_{\nu,diff}$) can be given as

$$Q_{\nu,diff} [\text{erg/cm}^3/\text{s}] \equiv \int \frac{\varepsilon_\nu n_\nu(\varepsilon_\nu)}{T_\nu^{diff}(\varepsilon_\nu)} d\varepsilon_\nu, \quad (34)$$

where the right-hand-side of Equation (34) is simply proportional to the leakage of the neutrino energy density : $\varepsilon_\nu n_\nu$ [erg/cm³] divided by the diffusion timescale T_ν^{diff} [s]. More details to estimate these quantities as well as how to determine the neutrino sphere are summarized in Appendix A.

Charged Current Interactions
$n\nu_e \leftrightarrow e^- p$
$p\bar{\nu}_e \leftrightarrow e^+ n$
$\nu_e A \leftrightarrow e^- A'$
Neutral Current Interactions
$\nu p \leftrightarrow \nu p$
$\nu n \leftrightarrow \nu n$
$\nu A \leftrightarrow \nu A$

Table 1: The opacity set included in the present simulation. Note that ν , in neutral current reactions, represents all species of neutrinos ($\nu_e, \bar{\nu}_e, \nu_x$) with ν_x representing heavy-lepton neutrinos (i.e. ν_μ, ν_τ and their anti-particles).

Striving for simplification of our modeling, we include a reduced set of neutrino-matter interactions (e.g., Table 1). Regarding the charged-current interactions, emission and absorption of electron neutrinos by neutrons (the first column in Table 1), emission and absorption of electron anti-neutrinos by proton (the second column), and emission and absorption of electron neutrinos by heavy nuclei (the third column), are included. For the neutral-current interactions, elastic scattering of all neutrino flavors off nucleons (the fourth and fifth column in Table 1), and the coherent elastic scattering (the sixth column) are included. For the cross sections of these reactions, we employ the ones summarized in Burrows et al. (2006) while omitting higher-order terms such as ion-ion correlations and weak magnetism for simplicity.

In computing the neutrino cooling rate ($Q_{\nu, intr}^C$), we furthermore consider the contribution from pair neutrino annihilation $Q_{e^-e^+\rightarrow\nu\bar{\nu}}$ (Cooperstein et al. 1986), plasmon decay $Q_{\gamma\rightarrow\nu\bar{\nu}}$ (Ruffert et al. 1996), and nucleon-nucleon bremsstrahlung $Q_{NN\rightarrow NN\nu\bar{\nu}}$ (Burrows et al. 2006), which are also summarized in Sekiguchi (2010). Hence $Q_{\nu, intr}^C$ can be expressed as,

$$Q_{\nu, intr}^C = Q_{e^-}^f + Q_{e^-}^h + Q_{e^+}^f + Q_{e^+}^h + \sum_{\nu \in (\nu_e, \bar{\nu}_e, \nu_x)} 2(Q_{e^-e^+\rightarrow\nu\bar{\nu}} + Q_{\gamma\rightarrow\nu\bar{\nu}} + Q_{NN\rightarrow NN\nu\bar{\nu}}), \quad (35)$$

where $Q_{e^-/+}^f$ and $Q_{e^-/+}^h$ represents the cooling rate by electron/positron capture on free nucleons and on heavy nuclei, respectively.

Concerning the neutrino heating ($Q^{\mu, H}$), we only include the dominant heating reactions in the gain region, which is absorption of electron/anti-electron neutrinos by free nucleons. Then $Q^{\mu, H}$ reads

$$Q^{\mu, H} = e^{-\beta\tau} \sum_{i \in (\nu_e, \bar{\nu}_e)} \int d\omega \kappa_{\omega, i} (-\mathcal{J}_{\omega, i} u^\mu - \mathcal{H}_{\omega, i}^\mu) \quad (36)$$

where ω denotes neutrino energy, $\kappa_{\omega, i}$ is the energy-dependent opacity for electron or anti-electron neutrinos (i.e. $i = \nu_e$ or $\bar{\nu}_e$ see Appendix A), and $\mathcal{J}_\omega, \mathcal{H}_\omega$ is the energy-dependent Eddington moments, respectively. Yielding to the prescription of the so-called light-bulb scheme (e.g., Nordhaus et al. (2010)), a term of $e^{-\beta\tau}$ is introduced to vanish the neutrino heating smoothly as the opacity becomes higher inward down to the neutrino sphere. To take a gray approximation, we replace the energy integration in Equation (36) with the root-mean-squared (RMS) energy of the streaming neutrinos ($\epsilon_{s\nu, i}$, see Appendix A for the definition) as

$$\int d\omega \kappa_{\omega, i} (-\mathcal{J}_{\omega, i} u^\mu - \mathcal{H}_{\omega, i}^\mu) \delta(\omega - \epsilon_{s\nu, i}) \longrightarrow (\epsilon_{s\nu, i})^2 \tilde{\kappa} (-\mathcal{J} u^\mu - \mathcal{H}^\mu), \quad (37)$$

where $\tilde{\kappa}$ denotes the monochromatic opacity, in which the energy-dependence is replaced with the one of the rms energy (namely, $\kappa_i = \tilde{\kappa}_i \cdot \epsilon_{s\nu, i}^2$)⁷. Since \mathcal{J}, \mathcal{H} in Equation (37) is related to the

⁷The delta function in the left-hand-side of Equation (37) may be replaced by the Fermi-Dirac function. In the case, an additional factor of $F_2(\eta_\nu, 0)$ (for the degeneracy limit $F_2(0, 0) \approx 2$) can be multiplied, which could potentially enhance the impacts of neutrino heating.

variables in the laboratory frame by Equations (28,29), the two-moment equations of E_ν and F_ν with the source terms are finally closed. Having given explicit forms of Q^μ , we are now moving on to summarize the GR hydrodynamic equations including the source terms in the next section.

2.4. Hydrodynamic Equations

From Equation (2), the hydrodynamic equations are written in a conservative form as,

$$\partial_t \rho_* + \partial_i (\rho_* v^i) = 0, \quad (38)$$

$$\begin{aligned} \partial_t \hat{S}_i + \partial_j (\hat{S}_i v^j + \alpha e^{6\phi} P \delta_i^j) &= -\hat{S}_0 \partial_i \alpha + \hat{S}_k \partial_i \beta^k + 2\alpha e^{6\phi} S_k^k \partial_i \phi \\ &\quad - \alpha e^{2\phi} (S_{jk} - P \gamma_{jk}) \partial_i \tilde{\gamma}^{jk} / 2 - e^{6\phi} \alpha Q^\mu \gamma_{i\mu}, \end{aligned} \quad (39)$$

$$\begin{aligned} \partial_t \hat{\tau} + \partial_i (\hat{S}_0 v^i + e^{6\phi} P (v^i + \beta^i) - \rho_* v^i) &= \alpha e^{6\phi} K S_k^k / 3 + \alpha e^{2\phi} (S_{ij} - P \gamma_{ij}) \tilde{A}^{ij} - \hat{S}_i D^i \alpha \\ &\quad + e^{6\phi} \alpha Q^\mu n_\mu, \end{aligned} \quad (40)$$

$$\partial_t (\rho_* Y_e) + \partial_i (\rho_* Y_e v^i) = \rho_* \Gamma_e, \quad (41)$$

where $\hat{X} \equiv e^{6\phi} X$, $\rho_* = \rho W e^{6\phi}$, $v^i = u^i / u^t$, $\hat{\tau} = \hat{S}_0 - \rho_*$, and Y_e is the electron fraction.

From Equations (32, 33), the source terms appearing in the right-hand-side of Equations (39, 40) can be explicitly written as,

$$\begin{aligned} -Q^\mu \gamma_{\mu i} &= -(Q^{\mu,C} - Q^{\mu,H}) \gamma_{\mu i} \\ &= - \sum_{\nu \in \nu_e, \bar{\nu}_e, \nu_x} \left[(1 - e^{-\beta_\nu \tau_\nu}) Q_{\nu, diff} + e^{-\beta_\nu \tau_\nu} Q_{\nu, intr}^C \right] u_i \\ &\quad + \sum_{\nu \in \nu_e, \bar{\nu}_e} e^{-\beta_\nu \tau_\nu} (\varepsilon_{s_\nu})^2 \tilde{\kappa}_\nu (-W F_{\nu i} + P_{\nu i}^k u_k) \end{aligned} \quad (42)$$

$$\begin{aligned} Q^\mu n_\mu &= (Q^{\mu,C} - Q^{\mu,H}) n_\mu \\ &= - \sum_{\nu \in \nu_e, \bar{\nu}_e, \nu_x} \left[(1 - e^{-\beta_\nu \tau_\nu}) Q_{\nu, diff} + e^{-\beta_\nu \tau_\nu} Q_{\nu, intr}^C \right] W \\ &\quad + \sum_{\nu \in \nu_e, \bar{\nu}_e} e^{-\beta_\nu \tau_\nu} (\varepsilon_{s_\nu})^2 \tilde{\kappa}_\nu (W E_\nu - F_\nu^k u_k). \end{aligned} \quad (43)$$

Γ_e in Equation (41) denotes the change in Y_e due to neutrino-matter interactions, which can be estimated in the same way as Q^ν . Given an appropriate EOS, the hydrodynamic equations (38)-(40) are closed. We employ a tabulated EOS by Shen et al. (98) for heavy nuclei and uniform nuclear matter. Since Shen EOS contains contributions only from baryons, we need to add contributions from electron/positron, and photon (see Appendix B for more details).

3. Initial Models and Numerical Methods

3.1. Initial Models

To assess GR and 3D effects on the neutrino-heating mechanism, we compute four models with a combination of SR or GR hydrodynamics in 1D or 3D, which we label as 1D-SR, 1D-GR, 3D-SR, and 3D-GR, respectively. We employ a widely used progenitor of a $15M_{\odot}$ star (Woosley & Weaver (1995), model “s15s7b2”). In our SR models, the space-time metric is assumed to be flat (i.e., $\alpha = 1$, $\beta^i = 0$, $\gamma_{ij} = \delta_{ij}$, $\phi = 0$, $K_{ij} = 0$) and the self-gravity is solved by the Poisson equation,

$$\nabla^2 \phi_{NT} = 4\pi S_0. \quad (44)$$

See Kuroda & Umeda (2010) for more details, such as about the methods to treat self-gravity in the AMR structure, and also to evolve the space-time metric with GR hydrodynamics.

To construct 1D models in our Cartesian code, the following condition for the spacial velocity u_i (and also for S_i) is enforced at every (hydro-) timestep,

$$u_i = \frac{x^j u_j}{|x|^2} x^i. \quad (45)$$

As can be read, this operation eliminates the non-radial components of the flow velocity and momentum. Although the artificial elimination could potentially lead to the shift of the kinetic energy into the thermal one, our 1D results (without and with neutrino heating/cooling) are in good agreement with the previous 1D results as will be mentioned in Appendix C and section 4. This suggests that the manipulation is not severely bad for the sake of this study.

The 3D computational domain consists of a cube of 10000^3 km³ volume in the Cartesian coordinates. In our 3D models, we set the maximum refinement AMR level (L_{AMR} , i.e., refine the AMR boxes in the vicinity of the center) at 5 in the beginning and then increment it as the collapse proceeds. We define the criterion to increment L_{AMR} every time the central density exceeds $10^{12,13,13.5}$ g cm^{−3} (see Kuroda & Umeda (2010) for more details). Each AMR level consists of 8^3 AMR blocks with a nested structure and each AMR block has 8^3 cubic cells. It thus yields an effective resolution of $\Delta x \sim 600$ m at bounce in the center of our 3D models.

3.2. Numerical Methods

Since the hydrodynamic and transport equations (Equations (38)-(40) and (21, 22)) are all expressed in a hyperbolic form, they can be evolved by a standard high-resolution-shock-capturing scheme. We utilize the HLL (Harten-Lax-van Leer) scheme (Harten et al. 1983) to evaluate the numerical fluxes. A reconstruction of the primitive variables defined at immediate left/right of the

cell surface is performed by a monotonized central method (Van Leer 1977). The fastest (or right-going) and slowest (or left-going) characteristic wave speeds of fluid system, λ_{flu} , for $i(\in x, y, z)$ direction are obtained by solving the following second order equation,

$$\mathcal{A}\lambda_{flu}^2 + 2\mathcal{B}\lambda_{flu} + \mathcal{C} = 0 \quad (46)$$

where

$$\begin{aligned} \mathcal{A} &= \left(\frac{1}{c_s^2} - 1\right)W^2 + 1, \\ \mathcal{B} &= \beta^i - \left(\frac{1}{c_s^2} - 1\right)(\alpha v^i - \beta^i)W^2, \\ \mathcal{C} &= \left(\frac{1}{c_s^2} - 1\right)W^2(\alpha v^i - \beta^i)^2 + \beta^{i2} - \alpha^2\gamma^{ii}, \end{aligned} \quad (47)$$

and c_s is the sound velocity (see Appendix B.3).

Meanwhile, the fastest and slowest characteristic wave speeds of radiation system, λ_{rad} , are assumed to have the same expression of the radiation pressure (Equation (23)) as

$$\lambda_{rad} = \frac{3\chi - 1}{2}\lambda_{rad,thin} + \frac{3(1 - \chi)}{2}\lambda_{rad,thick}, \quad (48)$$

where $\lambda_{rad,thin}$ and $\lambda_{rad,thick}$ is determined by P_{thin}^{ij} and P_{thick}^{ij} , respectively. According to Shibata et al. (2011), the fastest (slowest) wave speed in the optically thick or thin limit is evaluated by taking maximum (minimum) values, that is,

$$\left(-\beta^i + \frac{2W^2 p^i \pm \sqrt{\alpha^2\gamma^{ii}(2W^2 + 1) - 2W^2 p^{i2}}}{2W^2 + 1}, -\beta^i + p^i\right), \quad (49)$$

for the optically thick limit (where $p^i = \gamma^{ij}u_j/u^t$) and

$$\left(-\beta^i \pm \alpha \frac{F^i}{\sqrt{F_j F^j}}, -\beta^i + \alpha E \frac{F^i}{F_j F^j}\right), \quad (50)$$

for the optically thin limit, respectively. With these wave velocities regarding the fluid and radiation component ($\lambda_{flu/rad}$), we define the HLL flux (Anton et al. 2006) as

$$\mathbf{F}_{HLL} = \frac{\tilde{\lambda}_+ \mathbf{F}_L - \tilde{\lambda}_- \mathbf{F}_R + \tilde{\lambda}_- \tilde{\lambda}_+ (\mathbf{Q}_R - \mathbf{Q}_L)}{\tilde{\lambda}_+ - \tilde{\lambda}_-}, \quad (51)$$

where $\tilde{\lambda} = \lambda/\alpha$, L/R denotes the left/right states for the Riemann problem with $\mathbf{F}_{L/R}$ and $\mathbf{Q}_{L/R}$ representing the advection and conservative terms, respectively.

To ensure conservative laws at the interface of different AMR levels, we furthermore need to perform a “*refluxing*” procedure in estimating the numerical flux (see Kuroda & Umeda 2010). To

evolve the BSSN terms, we adopt the 4th order finite differencing for the spatial derivatives and the 4th order upwind differencing for the advection terms (Zlochower et al. 2005; Etienne et al. 2008). Numerical tests are presented in Appendix C, in which we first show a 1D adiabatic core-collapse test to validate the implementation of Shen EOS in the code, followed by the corresponding 1D tests including neutrinos.

4. Results

First let us compare prebounce features among the four models (1D-SR, 1D-GR, 3D-SR, and 3D-GR) in section 4.1, and move on to the postbounce phase in section 4.2. Then in section 4.3, we will discuss the 3D/GR effects on the neutrino-heating mechanism.

4.1. Infall and Bounce

We begin our comparisons with the infall, bounce, and immediate postbounce phase. As seen from Figure 2, collapse to bounce takes slightly less time in our GR models (137 ms) compared to the SR models (141 ms), and the central density ρ_c at bounce is approximately 2 times larger in the GR simulations than in the corresponding SR simulations (see the inset in Figure 2). For our non-rotating progenitor, the dynamics of collapsing iron core proceeds totally spherically till the stall of the bounce shock. This is the reason that the multi-D effects are invisible in the immediate postbounce phase. Hence we focus on the comparisons between the 1D-SR and 1D-GR model in the rest of this subsection.

Figure 3 shows several snapshots of the lepton fraction (Y_{total}), electron fraction (Y_e), and electron-type neutrino fraction (Y_{ν_e}) for the 1D-SR (left panel) and 1D-GR model (right panel), respectively. After neutrino trapping (i.e. at a central density of a few 10^{12} g cm $^{-3}$), the central lepton fraction (black lines) is shown to be conserved later on. In the trapped regions, the radial profile of the neutrino fraction (blue lines) is almost flat, while Y_{ν_e} shows a gradual increase to satisfy the β -equilibrium.

From Figure 4, it can be seen that the lepton fraction at bounce (right end-point in density) is slightly larger for the 1D-GR model (~ 0.364 , red line) compared to the 1D-SR model (~ 0.359 , black line). The slight suppression of deleptonization is possibly because the neutrino opacity is effectively enhanced because of the more compact core in the GR model. Note that this trend is qualitatively in accord with the previous 1D results in which a spectral neutrino transport was solved (e.g., Lentz et al. (2011); Müller et al. (2010)).

Figure 5 compares the mean energy of emergent neutrinos $\langle \varepsilon_\nu \rangle$ between the 1D-SR (left) and 1D-GR model (right) (see Appendix A for definition). The mean energy is shown to be 20% larger (maximally near at bounce) for the GR model compared to the SR counterpart. This is also

because of the more compact core due to the GR hydrodynamics (Figure 6), leading to a more hotter neutrino sphere at smaller radii.

4.2. 3D hydrodynamics in the postbounce phase

In the first 10 ms after bounce, the bounce shock turns into the so-called “passive” shock, which expands gradually outward with no positive radial velocities (Buras et al. 2006a). As shown in Figure 7, the average shock radii until the passive expansion starts ($t_{\text{pb}} \lesssim 10$ ms), remain almost close in all the models. It then diverges, which is more remarkable between the 3D models (red and blue lines). As will be discussed in the following, this is because multi-D effects (convection and SASI) sensitively affect the postshock hydrodynamic behaviors, also under the influence of the different treatment in gravity (SR versus GR).

First of all, let us compare the shock evolution among the 1D models in SR vs. GR (1D-SR (green line) or 1D-GR (black line)). As expected, the shock radius is generally more compact for the 1D-GR model (black line). On the other hand, the maximum shock extent is observed to be almost the same with each other ($\langle R_{\text{shock}} \rangle \sim 130$ km). Though we cannot unambiguously specify the reason, this trend was also seen in Müller et al. (2010), who compared the shock radii in 1D simulations with detailed neutrino transport between their GR model using the CFC approximation and the corresponding Newtonian model with the effective potential approach (e.g., right panel of their Figure 5). The maximum of $\langle R_{\text{shock}} \rangle$ in Figure 7 for our 1D models indicates the epoch when the passive expansion stops. Afterwards ($t_{\text{pb}} \gtrsim 70$ ms), the shock begins to shrink and a much more rapid recession is visible for the GR model (black line). The maximum shock radii and the shock recession timescale obtained here are again similar to those obtained in the previous 1D results for the same progenitor model (e.g., Lentz et al. (2011); Müller et al. (2010)).

Four snapshots in Figure 8 are helpful to characterize the postbounce features in our 3D-GR model. The top left panel is for $t_{\text{pb}} \approx 10$ ms, when the bounce shock stalls at a radius of ~ 90 km (seen as a central blueish sphere). From the sidewall panels, the dominance of the $\ell = 4$ and $m = 4$ mode can be seen in the postshock region, which is a numerical artifact inherent to the use of Cartesian coordinates. Comparing the top left to top right panel in Figure 8, the size of the outer sphere that marks the position of the shock (seen as greenish in the top right panel) becomes bigger, which is due to the passive expansion. At this stage, there forms the gain region in which neutrino heating dominates over neutrino cooling (e.g., Janka (2001)). The neutrino-driven convection gradually develops later on. The sidewall panels of the top right panel also indicate the growth of the postshock convection triggered by Rayleigh-Taylor instabilities. The entropy behind the standing shock becomes higher with time due to neutrino-heating, which can be inferred from a yellowish bubble in the bottom left panel. The high entropy bubbles ($s[k_B/\text{baryon}] \gtrsim 10$) rise and sink behind the standing shock. The shock deformation is dominated by unipolar and bipolar modes, which is a characteristic feature of the SASI. The size of the neutrino-heated regions grows bigger with time in a non-axisymmetric way, which is indicated by bubbly structures with increasing entropy (indicated by reddish regions in the bottom right panel).

During our simulation time (100 ms after bounce), the shock radii can reach much further out for our 3D-GR model (red line in Figure 7). In contrast, the shock has already shown a trend

of recession in other models. Before we focus on the reason of it in the final section, let us next compare the activities of convection and SASI that we only touched on above.

Figure 9 displays space-time diagrams of laterally averaged; (a) Brunt-Väisälä (BV) frequency ω_{BV} (top left panel) ; (b) the anisotropic velocity V_{aniso} (top right); (c) the pressure perturbation Δp in a logarithmic scale (bottom left), and (d) the net heating rate per baryon Q_{net} (bottom right) for our 3D-SR (top four panels) and 3D-GR models (the other four panels), respectively. Each of the quantities is defined as,

$$\omega_{\text{BV}} \equiv \text{sign}(C_{\text{L}}) \sqrt{|g_{\text{eff}} C_{\text{L}}|}, \quad (52)$$

where g_{eff} represents effective gravitational potential that is estimated by taking a radial gradient of the potential in the Newtonian limit ($g_{\text{eff}} = d\phi_{\text{NT}}/dr$). C_{L} is the Ledoux criterion;

$$C_{\text{L}} \equiv -\frac{\partial \rho}{\partial P} \bigg|_{s, Y_{\text{tot}}} \left(\frac{\partial P}{\partial s} \bigg|_{\rho, Y_{\text{tot}}} \frac{ds}{dr} + \frac{\partial P}{\partial Y_{\text{tot}}} \bigg|_{\rho, s} \frac{dY_{\text{tot}}}{dr} \right), \quad (53)$$

in which the neutrino contribution to entropy is taken into account where the β -equilibrium is satisfied (Buras et al. 2006a). Following Takiwaki et al. (2011), V_{aniso} is estimated as

$$V_{\text{aniso}} = \sqrt{\langle \rho [(v_r - \langle v_r \rangle)^2 + v_\theta^2 + v_\phi^2] \rangle / \langle \rho \rangle}, \quad (54)$$

where $\langle A \rangle$ represents the angle average of quantity A . We define the normalized pressure perturbation Δp and the net heating rate per nucleon Q_{net} as

$$\Delta p \equiv \frac{\sqrt{\langle p^2 \rangle - \langle p \rangle^2}}{\langle p \rangle}, \quad (55)$$

and

$$Q_{\text{net}} \equiv \frac{e^{6\phi} \alpha Q^\mu n_\mu}{\rho}, \quad (56)$$

respectively. At first glance of Figure 9, one may not see any big differences between the 3D-SR (top four panels) and 3D GR models (the other), but indeed there are. Let us first discuss the properties of the four panels (a) - (d) taking the 3D-SR model as a reference and then proceed to focus on the differences between SR and GR.

From panel (a) (top left) showing the BV frequency for the 3D-SR model, one can depict three typical convectively unstable regions in the postbounce phase; prompt convection (greenish region at $t_{\text{pb}} \lesssim 20$ ms behind the shock⁸, postshock convection (seen as a narrow horizontal stripe behind the shrinking shock (just behind the outer most boundary labeled by white line), and PNS

⁸Note that the shock is indicated by a white thin line quickly rising after bounce and the passive shock stalls at a radius of $R \sim 150$ km .

convection (clearly seen as a thick horizontal stripe above the PNS at a radius of $\sim 10 - 20$ km later than $t_{\text{pb}} \gtrsim 60$ ms).

In our 3D results, the PNS convection develops only very weakly before ~ 60 ms postbounce. This is due to the stabilizing effect by a positive entropy gradient (see the positive gradient persisting outside the PNS surface ($R \sim 10$ km) in the right panel of Figure 10). Afterwards, the PNS convection gradually becomes vigorous with time as the negative lepton gradient nascent the PNS becomes remarkable (see the steepening slope of Y_l near $R \sim 10 - 20$ km in the left panel of Figure 10). Comparing the black dotted line in Figure 10 (for 1D-GR model at $t_{\text{pb}} = 60$ ms) with the corresponding one (green line) for the 3D counterpart, the slope of the negative gradient is shown to become much smaller for the 3D model both in the profiles of lepton fraction (left panel) and entropy (right panel). This is a natural outcome of the convective overturns acting to wash out the local gradients.

From panel (b) in Figure 10 showing the anisotropic velocity, the postshock convection ($R \gtrsim 100$ km) is clearly seen as a reddish stripe running from top left to bottom right. Note in the panel that the prompt convection can be also seen like a narrow prolate spheroid colored by red at $t_{\text{pb}} \lesssim 20$ ms with $10 \lesssim R \lesssim 60$ km. As seen, convective overturns operate above the PNS ($\sim 10 - 20$ km in radius) and below the shock ~ 100 km in radius. In-between, the region with smaller anisotropic velocity is formed (seen as a horizontal stripe colored by deep-blue at a radius of $30 - 50$ km after $t_{\text{pb}} \sim 50$ ms). By comparing to panel (d) (the net heating rate) to panel (c), the region is overlapped with the cooling region ($Q_{\text{net}} < 0$). The smaller anisotropic velocity there is because the infalling velocities in the cooling layer are so high that the convectively unstable material cannot stay there for long. Such a configuration has been already presented in 2D (Buras et al. 2006a) and 3D results (Takiwaki et al. 2011).

Here let's see panel (c) not for the 3D-SR model but for the 3D-GR model for convenience. The accreting flows should receive an abrupt deceleration near at the bottom of the cooling layer (the dark colored region in panel (d) (bottom right)), below which the regions are convectively stable (panel (b)). There forms a strong pressure perturbation (seen as a greenish horizontal stripe near $R \sim 30$ km in panel (c)). Subsequently the pressure perturbations are seen to propagate outward before they hit the shock (panel (c), seen as up-going stripes to the shock), probably leading to the formation of the next vortices. These features are akin to the so-called advective-acoustic cycle (e.g., Fogliizzo & Tagger (2000); Fogliizzo (2002); Scheck et al. (2008) and references therein), which is also observed in our 3D-SR model (top four panels).

We now focus on the differences between the 3D-SR and 3D-GR models. Comparing the panel (a)'s between SR and GR, the unshocked core (regions below the PNS convection at $t_{\text{pb}} \gtrsim 50$ ms) is shown to be more compact for the GR model. Between the pair models, Figure 11 compares the maximum of the pressure perturbation that the advecting vortices form near in the vicinity of the

deceleration regions⁹. As seen, the pressure perturbation after bounce is generally larger for the GR model (red line) compared to the SR model (black line) in our simulation time. This is presumably because stronger gravitation pull in GR makes the position of the coupling radius deeper, leading to produce more energetic acoustic waves. It is not straightforward to say something very solid only from the figure, but what we observed in our 3D-GR model (i.e. generation of stronger acoustic waves and the largest shock extent compared to the SR counterpart) does not seem, at least, unfavorable to drive neutrino-driven explosions. In the next section, we move on to discuss more in detail how 3D and GR would potentially impact on the neutrino-heating mechanism.

4.3. 3D versus GR effects on the neutrino-heating mechanism

Recalling that the neutrino heating rate can be symbolically expressed as $Q_\nu^+ \propto L_\nu \langle \epsilon_\nu^2 \rangle$ (e.g., Janka (2001)), we first analyze the neutrino luminosities (L_ν) and the mean energies ($\langle \epsilon_\nu \rangle$) in the following. After that, we compare the dwell time to the neutrino-heating time in the gain region and discuss which one (3D-SR vs. 3D-GR) is most likely to satisfy the criterion to initial the neutrino-driven explosions.

Figure 12 shows evolution of the neutrino luminosities of all the species (for ν_e , ν_x (left panel), and $\bar{\nu}_e$ (right panel)) for all the computed models. Here the neutrino luminosity is calculated as

$$L_\nu \equiv \int \alpha e^{6\phi} Q^{\mu,C} n_\mu dx^3, \quad (57)$$

where $Q^{\mu,C}$ in Equation (32) takes into account all the cooling contributions.

The spike in the ν_e luminosity corresponds to the so-called neutronization, when the shock propagates out through the ν_e sphere. The peak ν_e luminosity for the GR models is $L_{\nu_e} \sim 3 \times 10^{53}$ erg s⁻¹ (insensitive to 1D or 3D), which is slightly luminous compared to those in the SR models ($L_{\nu_e} \sim 2.9 \times 10^{53}$ erg s⁻¹). Using the same progenitor (Woosley & Weaver 1995), this trend is qualitatively similar to Bruenn et al. (2001). On the other hand, recent studies in which more detailed weak interactions are included in the Boltzmann transport have shown that the peak ν_e luminosity becomes $\sim 10\%$ smaller for the GR models (e.g., Lentz et al. (2011); Müller et al. (2010)). This may carry an important message that the Boltzmann transport should be implemented in the full GR simulations to obtain a $\sim 10\%$ -order accuracy, which is not small at all when speaking about the neutrino-driven mechanism.

After the neutronization burst ($t_{\text{pb}} \sim 10$ ms), the ν_e luminosity for the GR models slightly increases later on, while it stays almost constant for the SR models during the simulation time (green and blue lines). The $\bar{\nu}_e$ luminosity after 50 ms postbounce (right panel in Figure 12) is highest for the 3D-GR model (red line), which is also the case for the ν_x luminosity (left panel). Although the luminosities change with time, the luminosities generally yield to the following order,

⁹Scheck et al. (2008) termed it as the "coupling radius" in which the coupling of vortices and acoustic waves occur.

for ν_e , 3D-GR > 1DGR, 3D-SR \sim 1D-SR,

for $\bar{\nu}_e$, 3D-GR > 1DGR, 3D-SR > 1D-SR,

for ν_x , 3D-GR > 1DGR, 3D-SR > 1D-SR.

To summarize, both 3D and GR work to raise the neutrino luminosities in the early postbounce phase. As seen from the left panel in Figure 12, GR maximally increases the ν_x luminosity up to $\sim 50\%$ (in 3D), while the maximum increase by 3D is less than $\sim 20\%$ (compare the $\bar{\nu}_e$ luminosity between the 3D-GR and 1D-GR model). These results indicate that compared to the spacial dimensionality, GR holds the key importance to enhance the neutrino luminosities.

Top two panels in Figure 13 compare the angle average of the RMS neutrino energy for ν_e (left panel) and $\bar{\nu}_e$ (right panel) after the break-out burst ($t_{\text{pb}} \gtrsim 10$ ms). As seen, the RMS energies are highest for the 1D-GR model (black line), followed in order by 1D-SR, 3D-GR, and 3D-SR. In accord with the previous 1D results (Lentz et al. 2011; Müller et al. 2010; Liebendörfer et al. 2004; Bruenn et al. 2001), our 3D results (albeit limited to the early postbounce phase) support the expectation that the neutrino RMS energies increase when switching from SR to GR hydrodynamics.

The reason for the higher neutrino energy is that the deeper gravitational well of GR produces more compact core structures, and thus hotter neutrino spheres at smaller radii. This is shown in the bottom panels in Figure 13 (compare the radii of the neutrino sphere between GR and SR models). The smaller neutrino energies for our 3D models compared to the corresponding 1D models (top panels) is due to their larger neutrino spheres (bottom panels). In our 3D models, the shock expands much further out assisted by convection and SASI (e.g., Figure 7), which also extends the positions of the neutrino spheres. The enlargement of the neutrino sphere in multi-D models is qualitatively consistent with the 2D post-Newtonian results by Buras et al. (2006a) including detailed neutrino transport.

As mentioned above, GR increases the neutrino luminosities and energies, while the 3D hydrodynamics works to make the neutrino energy smaller. What we like to discuss finally is whether the gain effects of GR could or could not overcome the possible loss effects of GR that should shorten the residency time of material in the gain region. And multi-D effects join in the game because they could potentially work against it to make the dwell time longer.

A widely prevailing indicator to diagnose the onset of the neutrino-driven explosions is the ratio of the residency timescale ($\langle t_{\text{res}} \rangle$) to the neutrino-heating timescale ($\langle t_{\text{heat}} \rangle$) in the gain region (e.g., Janka (2001); Thompson et al. (2005); Murphy & Burrows (2008))¹⁰. To estimate $\langle t_{\text{res}} \rangle$, we employ the effective advection timescale (Equation (8) in Buras et al. (2006a)), in which $\langle t_{\text{res}} \rangle$ is determined by the crossing time of mass shell between shock and gain radii. The local heating

¹⁰If this indicator is greater than unity, i.e. $\langle t_{\text{res}} \rangle / \langle t_{\text{heat}} \rangle > 1$, the neutrino heating proceeds fast enough to gravitationally unbind the fluid element, otherwise the matter is swallowed by the neutrino-cooling layer

timescale is estimated by the mass weighted average of the local heating timescale,

$$\tau_{\text{heat}} \equiv \frac{-\varepsilon_{\text{bind}}}{\dot{Q}_{\nu, \text{ total}}}, \quad (58)$$

where we obey the Newtonian expression to estimate the local binding energy as

$$\varepsilon_{\text{bind}} \equiv \rho \left(u^t \varepsilon + \frac{1}{2} v_i v^i + \phi_{NT} \right), \quad (59)$$

and the net neutrino heating rate is calculated by $\dot{Q}_{\nu, \text{ total}} \equiv e^{6\phi} \alpha Q^\mu n_\mu$ (see, Equation (40)). In estimating the heating timescale, the numerical cells that satisfy both $\varepsilon_{\text{bind}} < 0$ and $\dot{Q} > 0$ are only taken into account.

As seen from Figure 14, the shock revival seems most likely to occur for the 3D-GR model (red line) in our simulation time, which is followed in order by 3D-SR, 1D-SR and 1D-GR models. Thanks to a more degree of freedom, the residency timescale becomes much longer for the 3D models than for the 1D models. In addition, the increase of the neutrino luminosity and RMS energies due to GR (Figure 13) enhances the timescale ratio up to the factor of ~ 2 for the 3D-GR model (red line) compared to the SR counterpart (blue line). Therefore our results suggest that the combination of 3D and GR hydrodynamics could provide the most favorable condition to trigger the neutrino-driven explosions.

As expected from Figure 14, the shock revival will never occur afterwards for the 1D models that have already shown the sign of a rapid shock recession. On the other hand, the curves for the 3D models stay constant for the last 30 ms before our simulation terminates. For the $15 M_\odot$ progenitor employed in this paper, the neutrino-driven explosions are expected to take place later than ~ 200 ms postbounce at the earliest (Bruenn et al. 2010) and it could be delayed after ~ 600 ms postbounce (Marek & Janka 2009) as already mentioned. The parametric explosion models have shown that the earlier shock revival is good for making the explosion energy larger (e.g., Nordhaus et al. (2010)). The onset timescale of the neutrino-driven explosions predicted in 2D models (Marek & Janka 2009; Bruenn et al. 2010; Suwa et al. 2010, 2011) could be shorter if the combination effects of GR and 3D would have been included. We anticipate that this can be a possible remedy to turn the relatively underpowered 2D explosions into the powerful ones. To draw a robust conclusion, the energy and angle dependence of the neutrino transport should be accurately incorporated in our full GR simulations with the use of more detailed set of weak interactions. This work is only the very first step towards the climax to investigate these fascinating issues.

5. Summary

We presented the results from the first-generation multi-D core-collapse simulations in full GR that include an approximate treatment of neutrino transport. Using a M1 closure scheme with an analytic variable Eddington factor, we solved the energy-independent set of radiation energy

and momentum based on the Thorne’s momentum formalism. To simplify the source terms of the transport equations, a methodology of multiflavour neutrino leakage scheme was partly employed. Our newly developed code was designed to evolve the Einstein field equation together with the GR radiation hydrodynamic equations in a self-consistent manner while satisfying the Hamiltonian and momentum constraints. An adaptive-mesh-refinement technique implemented in the three-dimensional (3D) code enabled us to follow the dynamics starting from the onset of gravitational core-collapse of a $15 M_{\odot}$ star, through bounce, up to about 100 ms postbounce in this study. By computing four models that differ by 1D or 3D and by switching from SR to GR hydrodynamics, we studied how the spacial multi-dimensionality and GR would affect the dynamics in the early postbounce phase. Our 3D results support the anticipation in the previous 1D results that the neutrino luminosity and the average neutrino energy of any neutrino flavor in the postbounce phase generally increase when switching from SR to GR hydrodynamics. This is because the deeper gravitational well of GR produces more compact core structures, and thus hotter neutrino spheres at smaller radii. By analyzing the residency to the neutrino-heating timescale in the gain regions, we pointed out that the criteria to initiate neutrino-driven explosions could be most easily satisfied in the 3D models, irrespective of the SR or GR hydrodynamics. Keeping caveats in mind the omission of energy- and angle-dependence of the radiation fields and the use of reduced set of weak interactions in the present algorithm, our results indicated that the combination of 3D hydrodynamics and GR should provide the most favorable condition to drive a robust neutrino-driven explosion.

On top of the omission of the spectral and angle dependence of the neutrino transport, we think the most urgent task is to replace the leakage scheme with more realistic modeling of the source terms. In our 3D simulation, the numerical resolution behind the standing accretion shock is a few kilometers, which is not good enough to capture the growth of SASI accurately (Sato et al. 2009). The numerical viscosity is expected to be large especially in the vicinity of the shock, which may affect the growth of the SASI. It could also affect the growth of the turbulence in the postshock convectively active regions, which is very important to determine the success or failure of the neutrino-driven mechanism. To clearly see these effects of numerical viscosity, we need to conduct a convergence test in which a numerical gridding is changed in a parametric way (e.g. Hanke et al. (2011)).

The most up-to-date neutrino transport code in core-collapse supernova simulations can treat the multi-energy and multi-angle transport in 2D (Ott et al. 2008) and even in 3D simulations (Sumiyoshi & Yamada 2012) but mostly in the Newtonian hydrodynamics (see, however, Müller et al. (2011, 2012)). As was originally pointed out by Schwartz (1967) in the late 60’s, our exploratory results also support the importance of GR to draw a robust conclusion to the supernova mechanism, indeed. The combined effects of GR and 3D¹¹ should affect not only the supernova dynamics, but

¹¹It is worth mentioning that the MHD effects also remain to be studied (Kotake et al. (2004); Takiwaki et al. (2004, 2009); Burrows et al. (2007a); Guilet et al. (2011); Kuroda & Umeda (2010); Obergaulinger & Janka (2011);

also the observational signatures of gravitational-waves (e.g., Müller et al. (2011); Kotake et al. (2009a,b); Ott et al. (2011)), neutrino emission (e.g., Abbasi et al. (2011); Marek et al. (2009); Lund et al. (2010)), and explosive nucleosynthesis (e.g., Fujimoto et al. (2011); Thielemann et al. (2011)). Keeping our efforts to improve the caveats mentioned above, we are going to study these fascinating subjects one by one in the near future.

A. Determination of Neutrino Spheres

As illustrated in Figure 1, we have to calculate the neutrino optical-depth (τ_ν) to determine the position of the neutrino spheres. It can be done by solving the following differential equation (actually by a matrix inversion)

$$\frac{x^i D_i \tau_\nu}{r} = -\kappa_\nu, \quad (\text{A1})$$

with an appropriate boundary condition ($\tau_\nu|_{r \rightarrow \infty} = 0$). Here κ_ν represents the neutrino opacity of each species. Then the neutrino sphere is determined where the optical-depth exceeds $2/3$ ($\tau_\nu = 2/3$). For the matrix solver, we take the same one to solve the Poisson equation (44). As shown in Table 1, we only include a reduced, but the most fundamental set of weak interactions in the supernova cores, which consists of the charged-current interactions; $n\nu_e \leftrightarrow e^-p$, $p\bar{\nu}_e \leftrightarrow e^+n$, $\nu_e A \leftrightarrow e^-A'$ and scattering processes; $\nu p \leftrightarrow \nu p$, $\nu n \leftrightarrow \nu n$, $\nu A \leftrightarrow \nu A$. Note ν in the scattering processes, represents all species of neutrinos ($\nu_e, \bar{\nu}_e, \nu_x$). The opacity for electron, anti-electron, and heavy-lepton neutrinos can be expressed as

$$\kappa_{\nu_e} = \kappa_a(\nu_e n) + \kappa_a(\nu_e A) + \kappa_s(\nu_e n) + \kappa_s(\nu_e p) + \kappa_s(\nu_e A), \quad (\text{A2})$$

$$\kappa_{\bar{\nu}_e} = \kappa_a(\bar{\nu}_e p) + \kappa_s(\bar{\nu}_e n) + \kappa_s(\bar{\nu}_e p) + \kappa_s(\bar{\nu}_e A), \quad (\text{A3})$$

and

$$\kappa_{\nu_x} = \kappa_s(\nu_x n) + \kappa_s(\nu_x p) + \kappa_s(\nu_x A), \quad (\text{A4})$$

respectively (e.g., Ruffert et al. (1996)). Here the subindex a and s denote absorption and scattering processes, respectively. Detailed descriptions for the expressions of each opacity can be found in Bruenn (1985); Burrows et al. (2006).

Since we do not transfer the number density of neutrinos in the present scheme, we evaluate the emergent root-mean-squared (rms) energy of neutrinos $\varepsilon_{s_\nu, i}$ in the following way. Note here that the subscript $i = \nu_e, \bar{\nu}_e$ denotes the neutrino species. We first project the neutrino sphere defined in the cartesian grids to the spherical polar grids, which gives us the position of the neutrino sphere

Takiwaki & Kotake (2011), see also Kotake et al. (2006) for collective references therein).

expressed in the polar grids as $R_{\nu,i}(\theta, \phi)$. Then we identify $\varepsilon_{s_{\nu},i}$ with the energy of neutrinos at the neutrino sphere assuming that they stream freely outwards with possessing the information of the last scattering surface. Then $\varepsilon_{s_{\nu}}$ at arbitrary point (R, θ, ϕ) may be expressed as

$$\varepsilon_{s_{\nu},i}(R, \theta, \phi) \equiv \varepsilon_{\nu,i}(R_{\nu,i}(\theta, \phi), \theta, \phi). \quad (\text{A5})$$

Here $\varepsilon_{\nu,i}$ in the right-hand-side denotes the neutrino energy at $R_{\nu}(\theta, \phi)$, which is estimated by

$$\varepsilon_{\nu,i} = k_B T \frac{F_3(\eta_{\nu}, 0)}{F_2(\eta_{\nu}, 0)}, \quad (\text{A6})$$

where F_k is the Fermi-Dirac integral and $\eta_{\nu} = \mu_{\nu}/k_B T$ is the degeneracy parameter with μ_{ν} , T , k_B representing the neutrino chemical potential, matter temperature, and Boltzmann constant, respectively.

A.1. Neutrino diffusion terms

Here we briefly summarize how to determine the diffusion term;

$$Q_{\nu,\text{diff}} \equiv \int g_{\nu} \frac{\varepsilon_{\nu}^k n_{\nu}}{\alpha_{\nu} T_{\nu}^{\text{diff}}} d\varepsilon_{\nu} \quad (\text{A7})$$

according to Ruffert et al. (1996); Rosswog & Liebendörfer (2003); Sekiguchi (2010). In the above equation, depending on k ($=1$ or 0), one can obtain the energy (or number) diffusion rate. α_{ν} in Equation (A7) and β_{ν} in Equations (32)-(33) are model parameters that affect the neutrino diffusion timescale and the position of the neutrino spheres, respectively. We adjust these parameters ($\alpha_{\nu} = 2$ and $\beta_{\nu} = 3/2$) in such a way to fit the neutrino luminosity obtained in the 1D results using the IDSA scheme (see Appendix C.2). g_{ν} ($g_{\nu_e} = g_{\bar{\nu}_e} = 1$ and $g_{\nu_x} = 4$) in Equation (A7) simply represents a multiplicity of neutrino species. n_{ν} is the number density of neutrinos per each energy bin ε_{ν} in thermal equilibrium with matter and expressed by the Fermi-Dirac distribution function as

$$n_{\nu} = \frac{4\pi}{(hc)^3} \frac{\varepsilon_{\nu}^2}{1 + \exp(\frac{\varepsilon_{\nu} - \mu_{\nu}}{k_B T})}, \quad (\text{A8})$$

here μ_{ν} is the chemical potential of neutrinos. We define the diffusion time scale of neutrinos as

$$T_{\nu}^{\text{diff}} \equiv 3 \frac{\Delta x(\varepsilon_{\nu})}{c} \tau(\varepsilon_{\nu}) \quad (\text{A9})$$

where $\Delta x(\varepsilon_{\nu})$ is assumed to be $\Delta x(\varepsilon_{\nu}) = \tau(\varepsilon_{\nu})/\kappa(\varepsilon_{\nu})$. We assume the optical depth and opacity can be expressed as

$$\begin{aligned} \tau_{\nu}(\varepsilon_{\nu}) &\sim \varepsilon_{\nu}^2 \tilde{\tau}_{\nu} \\ \kappa_{\nu}(\varepsilon_{\nu}) &\sim \varepsilon_{\nu}^2 \tilde{\kappa}_{\nu} \end{aligned}$$

by neglecting the higher order correction terms of ε_ν . $\tilde{\tau}_\nu$ and $\tilde{\kappa}_\nu$ are energy independent optical depth and opacity, respectively. Finally, the energy integration in Eq.(A7) is rewritten as

$$\begin{aligned} Q_{\nu,\text{diff}} &= g_\nu \frac{1}{\alpha_\nu} \frac{4\pi c}{3(hc)^3} \frac{\tilde{\kappa}_\nu}{\tilde{\tau}_\nu^2} \int \frac{\varepsilon_\nu^k}{1 + \exp(\frac{\varepsilon_\nu - \mu_\nu}{k_B T})} d\varepsilon_\nu \\ &= g_\nu \frac{1}{\alpha_\nu} \frac{4\pi c}{3(hc)^3} \frac{\tilde{\kappa}_\nu}{\tilde{\tau}_\nu^2} (k_B T)^k F_k(\eta_\nu, 0) \end{aligned} \quad (\text{A10})$$

where $\eta_\nu = \mu_\nu/k_B T$ is the degeneracy parameter of neutrino.

B. Implementation of EOS

We employ Shen EOS (Shen et al. 1998) based on the Thomas-Fermi approximation and a minimization of the free energy within a relativistic mean field theory. The available data¹² is tabulated as a function of the three thermodynamic variables of density, temperature, and electron fraction as (ρ, T, Y_e) . We smoothly interpolate/extrapolate the original data as; $10^{3.1} \leq \rho \leq 10^{16} \text{ g cm}^{-3}$ with 200 equidistant intervals in a logarithmic scale, $10^6 \leq T \leq 10^{12} \text{ K}$ with 200 equidistant intervals in a logarithmic scale, and $0.01 \leq Y_e \leq 0.55$ with 50 equidistant intervals in a linear scale. The interpolation is performed first by a bicubic interpolation in the ρ - T plane and then by a cubic interpolation for Y_e direction.

Thermodynamic variables such as the total pressure, internal energy, and entropy reads,

$$P(\rho, T, Y_e) = P_b + P_{e^-} + P_{e^+} + P_\gamma, \quad (\text{B1})$$

$$\varepsilon(\rho, T, Y_e) = \varepsilon_b + \varepsilon_{e^-} + \varepsilon_{e^+} + \varepsilon_\gamma, \quad (\text{B2})$$

$$s(\rho, T, Y_e) = s_b + s_{e^-} + s_{e^+} + s_\gamma, \quad (\text{B3})$$

where subscripts b , e^- , and e^+ denote the contributions from baryon, electron, and positron, respectively.

B.1. Supplement to the original Shen EOS

Since the original Shen EOS contains contributions only from baryons, we need to add the remaining contributions from leptons and photons. Although it can be done straightforwardly by using formulae give in (e.g., Blinnikov et al. (1996)), we summarize them, for convenience, shortly in the following.

To construct the leptonic EOS, we have only to determine the electron chemical potential μ_{e^-} from a given data-set of proton fraction, density, and temperature (Y_p, ρ, T) . This can be done by

¹²We use the updated version which is obtained from <http://user.numazu-ct.ac.jp/sumi/eos/#shen2011>

the charge neutrality condition $Y_p = Y_e$, where Y_e is defined by

$$Y_e = \frac{n_{e^-} - n_{e^+}}{n_b}, \quad (\text{B4})$$

where $n_{e^-/+}$ and $n_b = \rho/m_u$ is the number density of electrons/positrons and bayrons with m_u being the atomic mass unit. $n_{e^-/+}$ can be expressed by

$$n_{e^-/+} = \sqrt{2} \frac{m_e c}{\hbar \pi^2} \beta^{3/2} \left[F_{1/2}(\eta_{e^-/+}, \beta) + \beta F_{3/2}(\eta_{e^-/+}, \beta) \right], \quad (\text{B5})$$

where $\beta \equiv k_B T / m_e c^2$ and $\eta^{-/+} \equiv \mu_{e^-/+} / k_B T$ with k_B , m_e , T , $\mu_{e^-/+}$ representing the Boltzmann constant, electron rest mass, temperature, and chemical potential of electron/positron, respectively (Blinnikov et al. 1996). $F_k(\eta, \beta)$ is the Fermi-Dirac integral,

$$F_k(\eta, \beta) = \int_0^\infty \frac{x^k (1 + \beta x/2)^{1/2}}{e^{x-\eta} + 1} dx, \quad (\text{B6})$$

where the useful analytical formulae of the integral (with their derivatives) are given in Tooper (1969); Miralles & van Riper (1996). Remembering that $\eta^- = -\eta^+$ is satisfied in the supernova cores due to high temperature ($\geq 10^9$ K), one can find the solution of μ_{e^-} by Equations (B4) and (B5). Thus the total pressure, specific internal energy, entropy (per nucleon) can be readily calculated as

$$P_e = \frac{2\sqrt{2}}{3} \frac{m_e^4 c^5}{\hbar^3 \pi^2} \beta^{5/2} \left[F_{3/2}(\eta_e, \beta) + \frac{\beta}{2} F_{5/2}(\eta_e, \beta) \right], \quad (\text{B7})$$

$$\varepsilon_e = \sqrt{2} \frac{m_e^2 c^3}{\hbar \pi^2} \beta^{5/2} \left[F_{3/2}(\eta_e, \beta) + \beta F_{5/2}(\eta_e, \beta) \right] \rho^{-1}, \quad (\text{B8})$$

$$s_e = \left[\frac{\rho \varepsilon_e + P_e - n_e \mu_e}{\rho T N_A k_B} \right], \quad (\text{B9})$$

where N_A is the Avogadro constant.

The contribution from photons is expressed as,

$$P_\gamma = \frac{1}{3} a_r T^4, \quad (\text{B10})$$

$$\varepsilon_\gamma = \frac{1}{\rho} a_r T^4, \quad (\text{B11})$$

$$s_\gamma = \frac{4 a_r T^4}{3 \rho T N_A k_B}, \quad (\text{B12})$$

where $a_r = 8\pi^5 k^4 / (15 c^3 h^3)$ denotes the radiation constant.

B.2. Primitive recovery

Since we evolve hydrodynamic equations in a conservative form, we need to obtain primitive variables from the conservative ones. For the primitive recovery, we first solve the following simultaneous equations to obtain $Z \equiv \rho h W^2$ and the Lorentz factor W (Cerdá-Durán et al. 2008; Kuroda & Umeda 2010)

$$(Z^2 - S^2)W^2 - Z^2 = 0 \quad (\text{B13})$$

$$\tau + D - Z + P(Z, W, Y_e/Y_l^t) = 0 \quad (\text{B14})$$

for a given conservative set of variables (ρ_*, S_i, τ) and the electron/total lepton fraction Y_e/Y_l^t . In the above equations, $S^2 \equiv \gamma^{ij} S_i S_j$ and $D \equiv \rho_*/e^{6\phi}$. P is the pressure and can be determined once the enthalpy $h = Z/DW$ and the rest mass density $\rho = D/W$ are given. We iteratively solve these equations by the Newton-Raphson method until the sufficient convergence is achieved.

B.3. The sound velocity

As given in Shibata & Sekiguchi (2005b), the sound velocity is expressed as,

$$c_s = \sqrt{\frac{1}{h} \left[\frac{\partial P}{\partial \rho} \Big|_{\varepsilon} + \frac{P}{\rho^2} \frac{\partial P}{\partial \varepsilon} \Big|_{\rho} \right]}, \quad (\text{B15})$$

where P and ε include the sum of contributions from baryon, electron, and photon. Regarding the partial derivatives of the thermodynamical variables, we take a finite differencing of Shen's EOS table for the baryonic part, meanwhile we use analytical formulae of the Fermi-Dirac integrals given in Miralles & van Riper (1996) for the leptonic sector.

C. Numerical Tests

C.1. Core-Collapse tests with Shen EOS

We first present the 1D (in the same manner as 1D-SR/GR models by neglecting the non-radial matter velocity and momentum) core-collapse run without neutrinos to validate the implementation of Shen EOS instead of the phenomenological one taken in the original code (Kuroda & Umeda 2010). In the case of the adiabatic collapse, the so-called prompt explosion is expected to occur for the $15M_{\odot}$ star (Woosley & Weaver 1995) as reported by Sumiyoshi et al. (2004) in their 1D GR Lagrangian simulations using the same EOS.

Figure 15 shows the profiles of density (left panel) and radial velocity (right panel) between the GR (solid line) and SR (dashed line) model, respectively.

As can be seen, the central density ρ_c (left panel) and the infall velocity (right panel) becomes higher for the GR model, which bounces at $\rho_c = 4.5 \times 10^{14} \text{ g cm}^{-3}$ with its inner-core baryon mass ($M_{\text{IC}} = 0.91 M_\odot$) being $\sim 0.1 M_\odot$ smaller compared to the SR counterpart. After bounce, the prompt shock propagates through the entire iron core (left panel in Figure 16) for both of the models. For the GR model, the shock reaches at a radius of 1000 km at ~ 20 ms after bounce, with its explosion energy in the range of $1 - 1.5 \times 10^{51} \text{ erg}$ (right panel in Figure 16), which is consistent with those obtained in Sumiyoshi et al. (2004).

As for another test to check numerical accuracy, we show the violation of the average Hamiltonian constraint C_{hm} and the Arnowitt-Deser-Misner mass (ADM mass) M_{ADM} in figure 17. We adopt the following form for C_{hm} (Shibata 2003)

$$C_{\text{hm}} = \frac{1}{M_{\text{bar}}} \int \frac{\rho_* \mathcal{H}}{\left[\left| \tilde{D}^i \tilde{D}_i e^\phi \right| + \left| \frac{e^\phi \tilde{R}}{8} \right| + \left| 2\pi(S_0 + E)e^{-\phi} \right| + \left| \frac{e^{5\phi}}{8} \left(\tilde{A}_{ij} \tilde{A}^{ij} - \frac{2}{3} K^2 \right) \right] } dx^3 \quad (\text{C1})$$

where M_{bar} is the proper rest mass and \mathcal{H} is the Hamiltonian constraint,

$$\mathcal{H} = \tilde{D}^i \tilde{D}_i e^\phi - \frac{e^\phi \tilde{R}}{8} + 2\pi(S_0 + E)e^{-\phi} + \frac{e^{5\phi}}{8} \left(\tilde{A}_{ij} \tilde{A}^{ij} - \frac{2}{3} K^2 \right) = 0. \quad (\text{C2})$$

M_{ADM} can be written as

$$M_{\text{ADM}} = \int \left[(S_0 + E)e^{-\phi} + \frac{e^{5\phi}}{16\pi} \left(\tilde{A}_{ij} \tilde{A}^{ij} - \frac{2}{3} K^2 - \tilde{\gamma}^{ij} \tilde{R}_{ij} e^{-4\phi} \right) \right] dx^3. \quad (\text{C3})$$

The violation of the ADM mass (red line) here for our 3D-GR model becomes larger in the post-bounce phase, which is not surprising considering the complicated non-linear nature of the field equation and also the presence of the shock that makes the accuracy of the high order shock-capturing scheme down to the first-order scheme inevitably. However, C_{hm} (black line) is generally kept less than 10^{-3} in the postbounce phase, which seems to be slightly better than the typical ones obtained in the previous full GR simulations (e.g., Shibata (2003); Sekiguchi (2010)).

C.2. Tests for Transport Scheme

For numerical tests of our transport algorithm, we present the check for the trapped and streaming neutrinos, respectively. Note again that the sum of the streaming and trapped neutrinos is transported by the evolution equations (Equations (21), (22)). The streaming part can be estimated by subtracting the trapped contribution from the sum, because the trapped part can be simply determined by local hydrodynamic quantities (i.e. density, temperature, and Y_e).

Figure 18 shows comparison of the RMS neutrino energy of the trapped neutrino and the one obtained by the IDSA scheme below the neutrino sphere. As seen, the assumption of β -equilibrium condition works well in the high density region and both of them show a quite similar profile there,

in which neutrinos are essentially trapped by matter. The agreement regarding the position of the neutrino sphere also certifies our estimate of the RMS neutrino energy (Equation A5).

To check the properties of the streaming neutrinos, which is very relevant in the postbounce dynamics, we check the following two points, which are (1) whether the radiation energy flux falls with proportional to r^{-2} above the neutrino sphere and (2) whether the gain region, in which the net heating rate becomes positive, can be formed similar to previous studies. Figure 19 shows radial profiles of the radiation (energy) flux that is obtained by solving the closed set of the two-moments equations (Equations (21,22)).

As seen, the energy fluxes change with r^{-2} (compare with the black line) irrespective of neutrino species outside the neutrino spheres. Note that the position of the neutrino spheres can be seen as the intersection point between the horizontal line ($\tau = 2/3$) and the solid lines. The radii of the neutrino spheres obeys a canonical order $R_{\nu_x} < R_{\bar{\nu}_e} < R_{\nu_e}$. The emergent neutrino energy flux can be estimated by $4\pi r^2, 4\pi r^2 \times (F_{r,\nu_e}, F_{r,\bar{\nu}_e}, F_{r,\nu_x}) \sim (9 \times 10^{52}, 8 \times 10^{52}, 4 \times 10^{52}) \text{ erg s}^{-1}$, which are all in good agreement with the luminosity defined by Equation (57) as plotted in Figure 12.

Figure 20 shows evolution of the net heating rate and radial velocity along the x axis for our 3D-GR model (see, Sec. 3) at selected postbounce epochs. As the passive shock propagates (from top left to bottom right panels), the gain region also gets larger. This reflects that the neutrino absorption on free nucleons predominantly takes place in the (enlarging) postshock region. The positive peak in the net heating rate is shown to be around 0.2 GeV/nuc/s in the first 100 ms postbounce, which is in agreement with those in previous studies (e.g., Liebendörfer et al. (2001); Sumiyoshi et al. (2005); Ott et al. (2008)).

REFERENCES

- Abbasi, R., Abdou, Y., Abu-Zayyad, T., et al. 2011, A&A, 535, A109
- Alcubierre, M. & Brügmann, B., 2001, Phys. Rev. D, 63, 104006
- Antón, L., Zanotti, O., Miralles, J. A., Martí, J. M., Ibáñez, J. M., Font, J. A., & Pons, J. A., 2006, ApJ, 637, 296
- Audit, E., Charrier, P., Chièze, J. -, & Dubroca, B. 2002, arXiv:astro-ph/0206281
- Baumgarte, T. W., & Shapiro, S. L., 1999, Phys. Rev. D, 59, 024007
- Bethe, H. A. & Wilson, J. R., 1985, ApJ, 295, 14
- Blondin, J. M., Mezzacappa, A. & DeMarino, C., 2003, ApJ, 584, 971
- Blinnikov, S. I., Dunina-Barkovskaya, N. V., & Nadyozhin, D. K. 1996, ApJS, 106, 171
- Bruenn, S. W., De Nisco, K. R., & Mezzacappa, A., 2001, ApJ, 560, 326

- Bruenn, S. W., Mezzacappa, A., Hix, W. R., Blondin, J. M., Marronetti, P., Messer, O. E. B., Dirk, C. J., & Yoshida, S. 2010, ArXiv e-prints
- Buras, R., Rampp, M., Janka, H.-Th. & Kifonidis, K., 2006a, A&A, 447, 1049
- Buras, R., Janka, H.-Th., Rampp, M. & Kifonidis, K., 2006b, A&A, 457, 281
- Burrows, A., Hayes, J. & Fryxell, B. A., 1995, 450, 830
- Burrows, A., Livne, E., Dessart, L., Ott, C. D. & Murphy, J., 2006, ApJ, 640, 878
- Burrows, A., Reddy, S., & Thompson, T. A. 2006, Nuclear Physics A, 777, 356
- Burrows, A., Dessart, L., Livne, E., Ott, C. D., & Murphy, J. 2007a, Astrophys. J., 664, 416
- Bruenn, S. W. 1985, ApJS, 58, 771
- Cerdá-Durán, P., Font, J. A., Antón, L., & Müller, E., 2008, A&A, 492, 937
- Colgate, S. A. & White, R. H., 1966, ApJ, 143, 626
- Cordero-Carrión, I., Cerdá-Durán, P., Dimmelmeier, H., et al. 2009, Phys. Rev. D, 79, 024017
- Cooperstein, J., van den Horn, L. J. & Baron, E. A., 1986, ApJ, 309, 653
- Duez, M. D., Liu, Y. T., Shapiro, S. L., Shibata, M. & Stephens, B. C., 2006, Phys. Rev. D, 73, 104015
- Dimmelmeie, H., Font, J. A. & Müller, E., 2002, A&A, 388, 917
- Epstein, R. I., & Pethick, C. J. 1981, ApJ, 243, 1003
- Etienne, Z. B., Faber, J. A., Liu, Y. T., Shapiro, S. L., Taniguchi, K., Baumgarte, T. W., 2008, Phys. Rev. D, 77, 084002
- Müller, E., Janka, H. ., & Wongwathanarat, A. 2011, ArXiv e-prints
- Fernández, R. & Thompson, C. 2009a, ApJ, 703, 1464
- . 2009b, ApJ, 697, 1827
- Foglizzo, T. & Tagger, M., 2000, A&A, 363, 174
- Foglizzo, T., A&A, 392, 353
- Foglizzo, T., Scheck, L. & Janka, H.-Th., 2006, ApJ, 652, 1436
- Fujimoto, S.-i., Kotake, K., Hashimoto, M.-a., Ono, M., & Ohnishi, N. 2011, ApJ, 738, 61

- Thielemann, F.-K., Arcones, A., Käppeli, R., et al. 2011, *Progress in Particle and Nuclear Physics*, 66, 346
- Fuller, G., Fowler, W. A. & Newman, M., 1985, *AJ*, 293, 1
- Guilet, J., Foglizzo, T., & Fromang, S. 2011, *ApJ*, 729, 71
- Fryer, C. L. 2004, *Astrophys. J. Lett.*, 601, L175
- Fryer, C. L., Holz, D. E., & Hughes, S. A. 2002, *Astrophys. J.*, 565, 430
- Hanke, F., Marek, A., Mueller, B., & Janka, H.-T. 2011, arXiv:1108.4355
- Harten, A., Lax, P. & van Leer, B., 1983, On upstream differencing and Godunov type methods for hyperbolic conservation laws. *SIAM review*. 25(1), pp 35-61
- Heger, A., Fryer, C. L., Woosley, S. E., Langer, N. & Hartmann, D. H., 2003, *ApJ*, 591, 288
- Herant, M., Benz, W. & Colgate, S., 1992, *ApJ*, 395, 642
- Iwakami, W., Kotake, K., Ohnishi, N., Yamada, S., Sawada, K., 2008, *ApJ*, 678, 1207
- . 2009, *Astrophys. J.*, 700, 232
- Janka, H.-Th., 2001, *A&A*, 368, 527
- Janka, H.-T. & Müller, E., 1996, *A&A*, 306, 167
- Kitaura, F. S., Janka, H.-Th. & Hillebrandt, W., 2006, *A&A*, 450, 345
- Kuroda, T. & Umeda, H., 2010, *ApJS*, 191, 439
- Kotake, K., Yamada, S., & Sato, K. 2003, *ApJ*, 595, 304
- Kotake, K., Sawai, H., Yamada, S., & Sato, K. 2004, *Astrophys. J.*, 608, 391
- Kotake, K., Sato, K., & Takahashi, K. 2006, *Reports of Progress in Physics*, 69, 971
- Kotake, K., Iwakami, W., Ohnishi, N., & Yamada, S. 2009, *ApJ*, 697, L133
- Kotake, K., Iwakami, W., Ohnishi, N., & Yamada, S. 2009, *ApJ*, 704, 951
- Kotake, K. 2011, arXiv:1110.5107, accepted to *Comptes Rendus Physique*
- Lentz, E. J., Mezzacappa, A., Bronson Messer, O. E., et al. 2011, arXiv:1112.3595
- Levermore, C. D., 1984, *Journal of Quantitative Spectroscopy and Radiative Transfer*, 31, 149
- Lieboldörfer, M., Mezzacappa, A., Thielemann, F.-K., Messer, O. E., Hix, W. R. & Bruenn, S. W., 2000, *Phys. Rev. D*, 63, 103004

- Liebendörfer, M., Mezzacappa, A., Thielemann, F.-K., et al. 2001, *Phys. Rev. D*, 63, 103004
- Liebendörfer, M., Messer, O. E. B., Mezzacappa, A., et al. 2004, *ApJS*, 150, 263
- Liebendörfer, M., Rampp, M., Janka, H.-Th., & Mezzacappa, A., 2005, *ApJ*, 620, 840
- Liebendörfer, M. 2005, *ApJ*, 633, 1042
- Liebendörfer, M., Whitehouse, S. C., & Fischer, T. 2009, *ApJ*, 698, 1174
- Lindquist, R. W. 1966, *Annals of Physics*, 37, 487
- Lund, T., Marek, A., Lunardini, C., Janka, H.-T., & Raffelt, G. 2010, *Phys. Rev. D*, 82, 063007
- Obergaulinger, M. & Janka, H.-T. 2011, *ArXiv e-prints*
- May, M. M., & White, R. H. 1966, *Physical Review*, 141, 1232
- Maeda, K., et al. 2008, *Science*, 319, 1220
- Marek, A. & Janka, H.-Th., 2009, *ApJ*, 694, 664
- Marek, A., Janka, H.-T., Müller, E. 2009, *A&A*, 496, 475
- Mezzacappa, A., & Matzner, R. A. 1989, *ApJ*, 343, 853
- Müller, E., Janka, H.-T. & Wongwathanarat, A., 2011, *A&A*, in press
- Misner, C. W., & Sharp, D. H. 1964, *Physical Review*, 136, 571
- Miralles, J. A. & van Riper, K. A., 1996, *ApJS*, 105, 407
- Mueller, B., Janka, H.-T., Marek, A., et al. 2011, *arXiv:1112.1913*
- Müller, B., Janka, H.-T., & Dimmelmeier, H. 2010, *ApJS*, 189, 104
- Mueller, B., Janka, H.-T., & Marek, A. 2012, *arXiv:1202.0815*
- Murphy, J. W., & Burrows, A., 2008, *ApJ*, 688, 1159
- Nomoto, K. & Mashimoto, M. 1988, *Phys. Rep.*, 163, 13
- Nordhaus, J., Burrows, A., Almgren, A. & Bell, J., 2010, *ApJ*, 720, 694
- Ohnishi, N., Kotake, K. & Yamada, S., 2006, *ApJ*, 641, 1018
- . 2007, *ApJ*, 667, 375
- Ott, C. D., Dimmelmeier, H., Marek, A., et al. 2007, *Physical Review Letters*, 98, 261101
- Ott, C. D., Burrows, A., Dessart, L., & Livne, E. 2008, *ApJ*, 685, 1069

- Ott, C. D., O'Connor, E. P., & Dasgupta, B. 2011, arXiv:1111.6282
- Ott, C. D., Reisswig, C., Schnetter, E., et al. 2011, *Physical Review Letters*, 106, 161103
- Sato, J., Foglizzo, T., & Fromang, S. 2009, *ApJ*, 694, 833
- Scheck, L., Plewa, T., Janka, H.-T., Kifonidis, K., & Müller, E., 2004, *Phys. Rev. Lett.*, 92, 011103
- Scheck, L., Kifonidis, K., Janka, H., & Müller, E. 2006, *A&A*, 457, 963
- Scheck, L., Janka, H.-T., Foglizzo, T. & Kifonidis, K., 2008, *A&A*, 477, 931
- Shen, H., Toki, H., Oyamatsu, K. & Sumiyoshi, K., 1998, *Nucl. Phys. A*, 637, 435
- Schwartz, R. A. 1967, *Annals of Physics*, 43, 42
- Sekiguchi, Y., 2010, *Progress of Theoretical Physics*, 124, 331
- Shibata, M., & Nakamura, T., 1995, *Phys. Rev. D*, 52, 5428
- Shibata, M. 2003, *Phys. Rev. D*, 67, 024033
- Shibata, M., & Sekiguchi, Y., 2005, *Phys. Rev. D*, 71, 024014
- Shibata, M., & Sekiguchi, Y., 2005, *Phys. Rev. D*, 72, 044014
- Shibata, M., Kiuchi, K., Sekiguchi, Y. & Suwa, Y., 2011, *Progress of Theoretical Physics*, 125, 1255
- Sumiyoshi, K., Suzuki, H., Yamada, S., & Toki, H. 2004, *Nuclear Physics A*, 730, 227
- Sumiyoshi, K., Yamada, S., Suzuki, H., Shen, H., Chiba, S. & Toki, H., 2005, *ApJ*, 629, 922
- Sumiyoshi, K., Yamada, S., Suzuki, H., et al. 2005, *ApJ*, 629, 922
- Sumiyoshi, K., Yamada, S., & Suzuki, H. 2007, *ApJ*, 667, 382
- Sumiyoshi, K., & Yamada, S. 2012, arXiv:1201.2244
- Suwa, Y., Kotake, K., Takiwaki, T., Whitehouse, S. C., Liebendörfer, M. & Sato, K., 2010, *PASJ*, 62, L49
- Suwa, Y., Kotake, K., Takiwaki, T., Liebendörfer, M., & Sato, K. 2011, *ApJ*, 738, 165
- Takiwaki, T., Kotake, K., Nagataki, S., & Sato, K. 2004, *ApJ*, 616, 1086
- Takiwaki, T., Kotake, K., & Sato, K. 2009, *Astrophys. J.*, 691, 1360
- Takiwaki, T., Kotake, K. & Suwa, Y., 2011, arXiv:1108.3989, accepted to *ApJ*
- Takiwaki, T., & Kotake, K. 2011, *ApJ*, 743, 30

- Tanaka, M., Kawabata, K. S., Maeda, K., Iye, M., Hattori, T., Pian, E., Nomoto, K., Mazzali, P. A., & Tominaga, N., 2009, *ApJ*, 699, 1119
- Thompson, T. A., Burrows, A., & Pinto, P. A. 2003, *Astrophys. J.*, 592, 434
- Thompson, T. A., Quataert, E., & Burrows, A. 2005, *ApJ*, 620, 861
- Thorne, K. S. 1981, *MNRAS*, 194, 439
- Tooper, R. F. 1969, *ApJS*, 156, 1075
- Rampp, M. & Janka, H.-Th., 2000, *ApJ*, 539, L33
- Rosswog, S. & Liebendörfer, M., 2003, *MNRAS*, 342, 673
- Ruffert, M., Janka, H.-Th. & Schäfer, G., 1996, *A&A*, 311, 532
- van Riper, K. A. 1979, *ApJ*, 232, 558
- van Riper, K. A., & Lattimer, J. M. 1981, *ApJ*, 249, 270
- van Riper, K. A. 1982, *ApJ*, 257, 793
- Van Leer, B., 1977, *Journal of Computational Physics*, 23, 263
- Wang, L., Howell, D. A., Höflich, P., & Wheeler, J. C. 2001, *Astrophys. J.*, 550, 1030
- Wilson, J. R. 1971, *ApJ*, 163, 209
- Wilson, J. R., 1985, *Numerical Astrophysics*, p422
- Woosley, S. E. & Weaver, T. A., 1995, *ApJS*, 101, 181
- Woosley, S. E., Heger, A., & Weaver, T. A. 2002, *Reviews of Modern Physics*, 74, 1015
- Yamada, S. 1997, *ApJ*, 475, 720
- Yamada, S., Janka, H.-T., & Suzuki, H. 1999, *A&A*, 344, 533
- Yakunin, K. N., Marronetti, P., Mezzacappa, A., Bruenn, S. W., Lee, C., Chertkow, M. A., Hix, W. R., Blondin, J. M., Lentz, E. J., Bronson Messer, O. E., & Yoshida, S. 2010, *Classical and Quantum Gravity*, 27, 194005
- Zlochower, Y., Baker, J. G., Campanelli, M. & Lousto, C. O., 2005, *Phys. Rev. D*, 72, 024021

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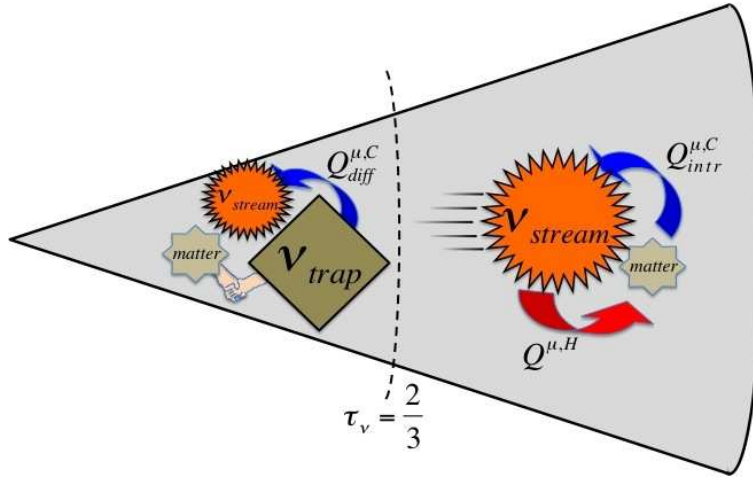


Fig. 1.— A schematic illustration how to model the source terms in our transport scheme. In this figure, trapped and streaming neutrinos are represented by “ ν_{trap} ” and “ ν_{stream} ”, respectively. $Q^{\mu,C}_{\text{diff}}$, $Q^{\mu,C}_{\text{intr}}$, and $Q^{\mu,H}$ denotes the coupling terms between them (see text for more details).

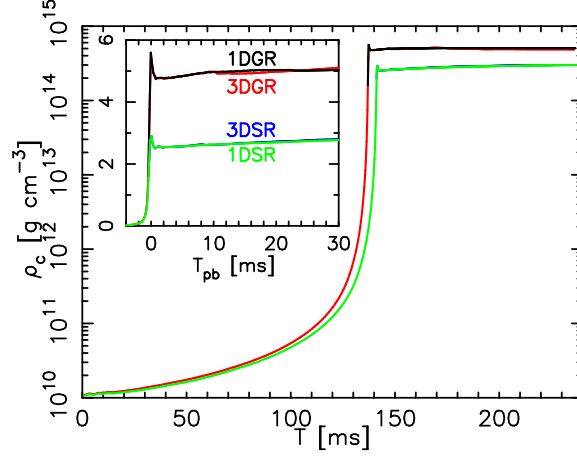


Fig. 2.— Central density ρ_c as a function of time from initial collapse for our models of 3D-GR (red), 1D-GR (black), 3D-SR (blue), and 1D-SR (green), respectively. The inset is just zoom up near bounce, in which the time is measured from bounce ($t_{\text{pb}} \equiv 0$) and the vertical lines represent the central density normalized by $10^{14} \text{ g cm}^{-3}$ in a linear scale.

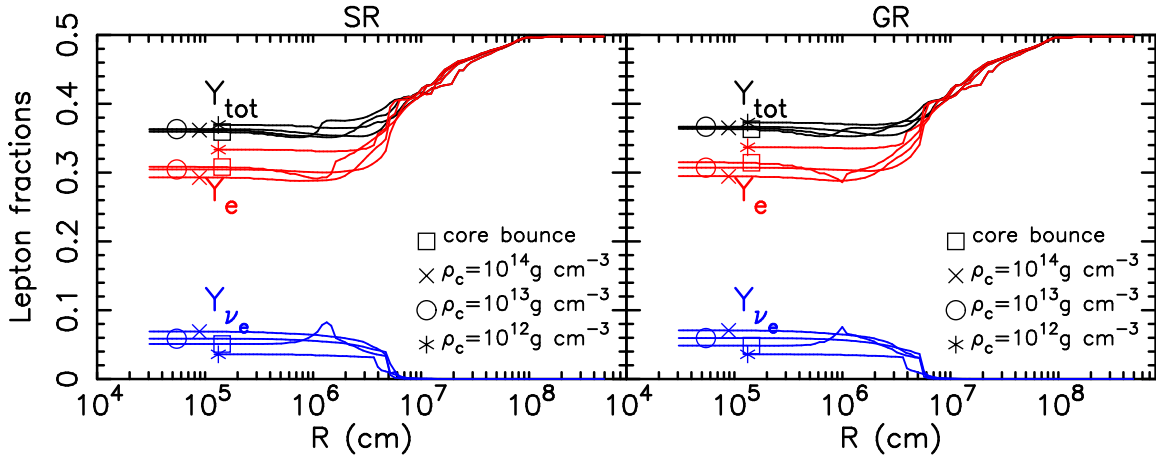


Fig. 3.— Profiles of the total lepton fraction (Y_{total}), electron fraction (Y_e), and electron-type neutrino fraction (Y_{ν_e}) for the 1D-SR (left panel) and 1D-GR model (right panel) at times, when the central density reaches the value as indicated in the plots.

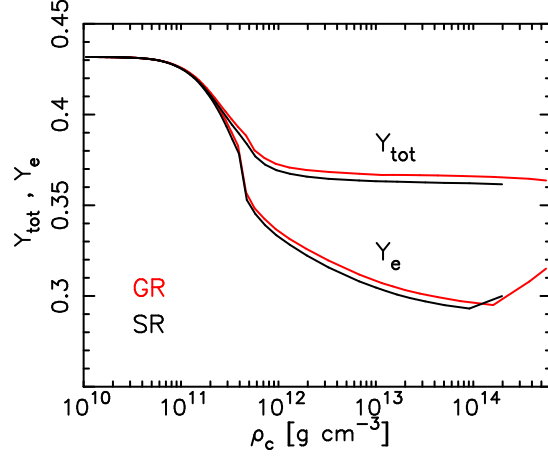


Fig. 4.— Comparison of the electron and lepton fraction versus central density during collapse for the 1D-SR (black line) and 1D-GR (red line) model, respectively.

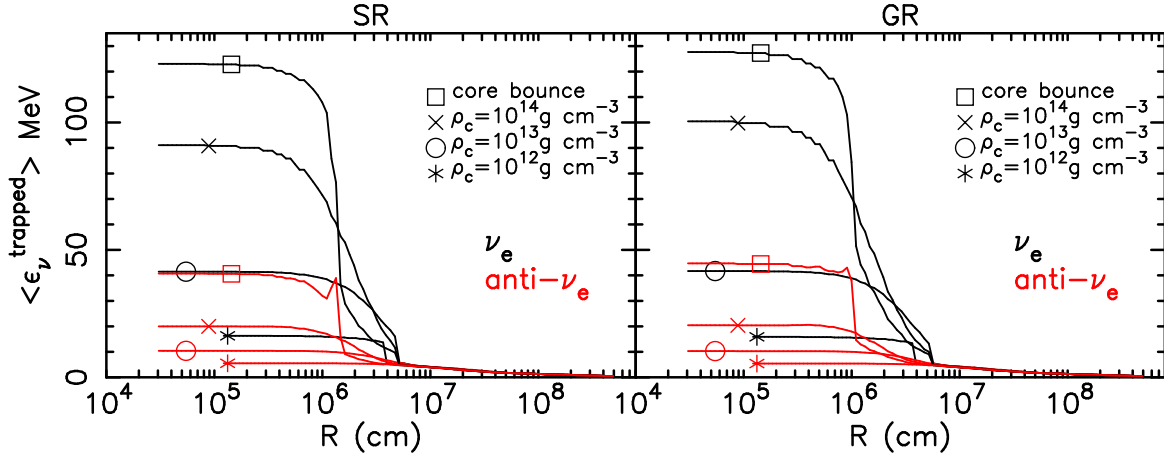


Fig. 5.— Same as Figure 3, but for the profiles of the RMS neutrino energies for ν_e (black) and $\bar{\nu}_e$ (red).

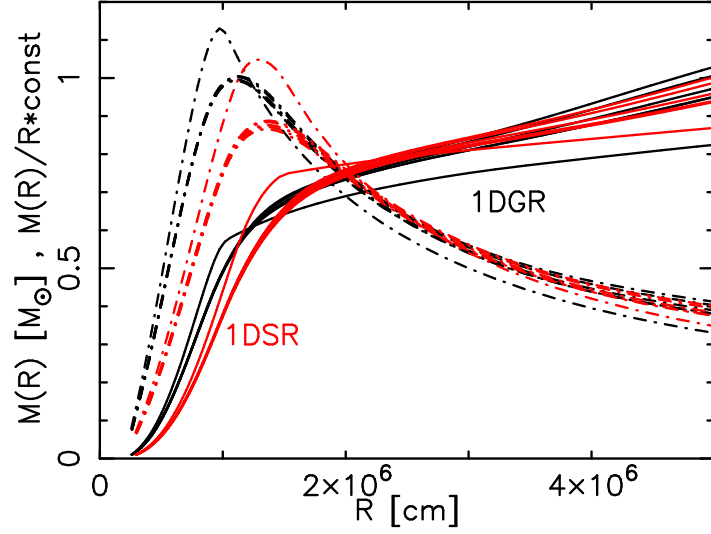


Fig. 6.— Profiles of enclosed mass $M(R)$ (*solid*) and the compactness parameter $M(R)/R$ (*dash-dotted*) as a function of radius R , in which lines are drawn every 2 ms in the first 10 ms postbounce for the 1D-SR (red line) and 1D-GR (black line) model, respectively. The 1D-GR model has the maximum compactness parameter that is 10% larger, and the mass of the homologous core at bounce that is 20 % smaller compared to the 1D-SR model.

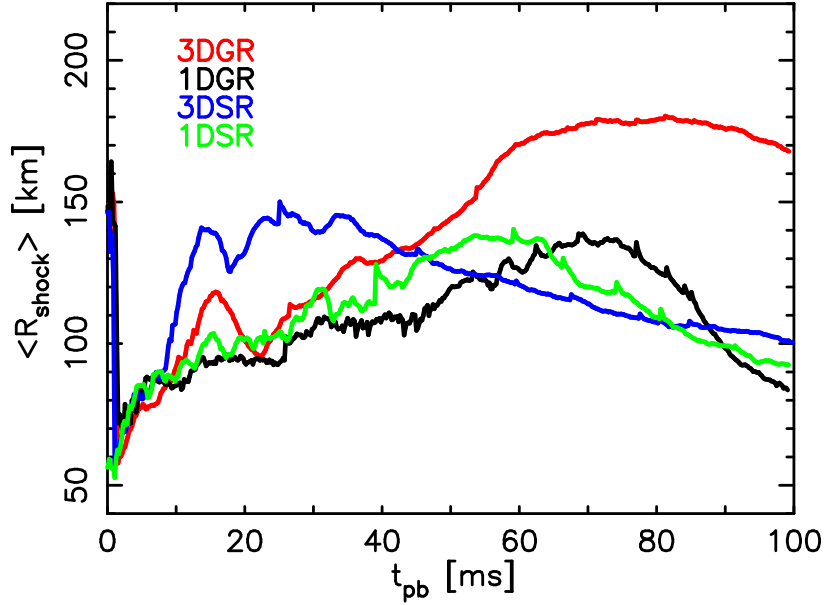


Fig. 7.— Evolutions of average shock radii as a function of post-bounce time t_{pb} for the four variant models.

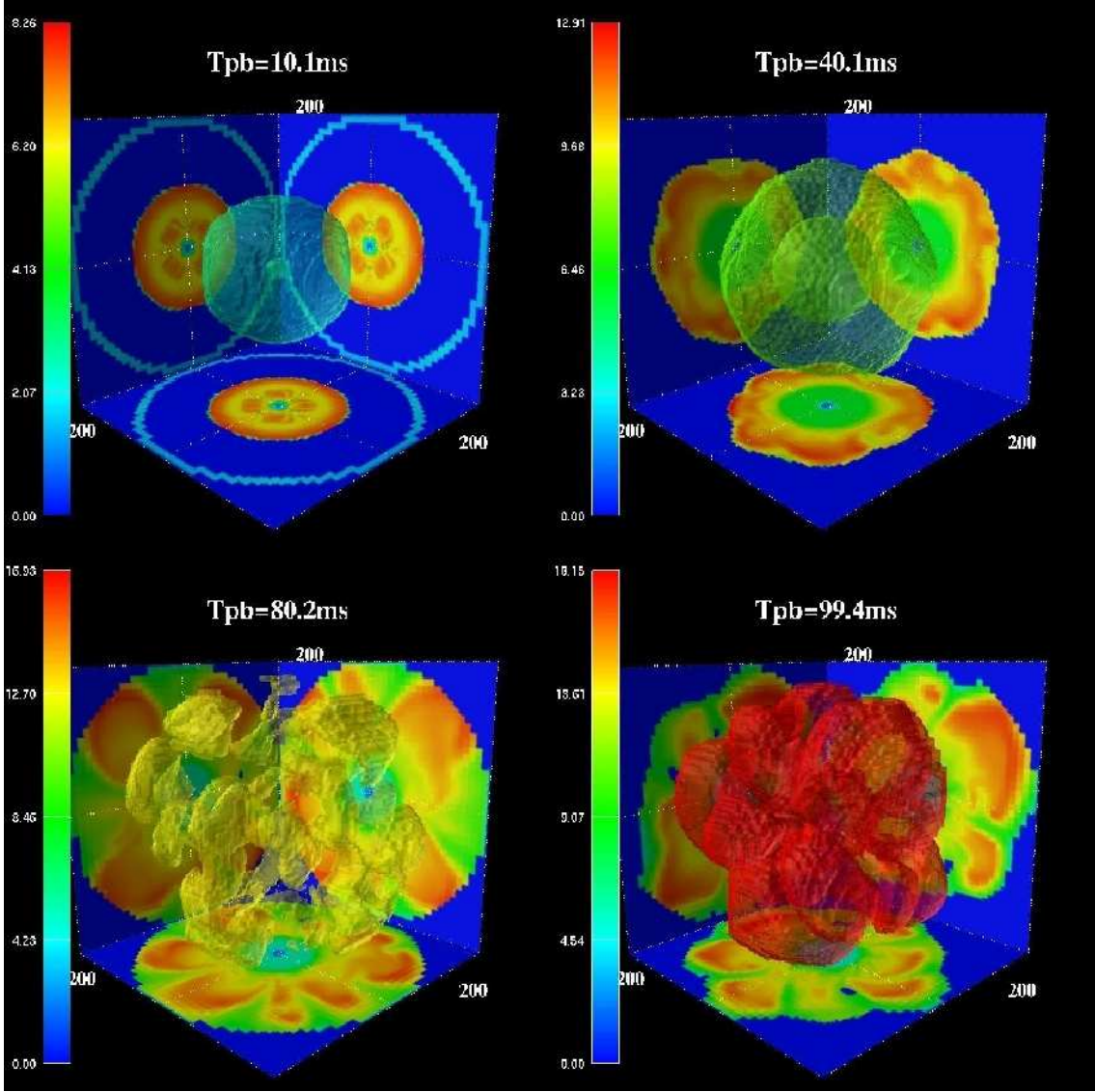


Fig. 8.— Three dimensional plots of entropy per baryon for four snapshots (top left; $t_{pb} = 10$ ms, top right; $t_{pb} = 40$ ms, bottom left; $t_{pb} = 80$ ms, and bottom right; $t_{pb} = 100$ ms) for the 3D-GR model. The contours on the cross sections in the $x = 0$ (back right), $y = 0$ (back bottom), and $z = 0$ (back left) planes are, respectively projected on the sidewalls of the graphs to visualize 3D structures. For each snapshot, the arbitrary chosen iso-entropy surface is shown, and the linear scale is indicated along the axis in unit of km.

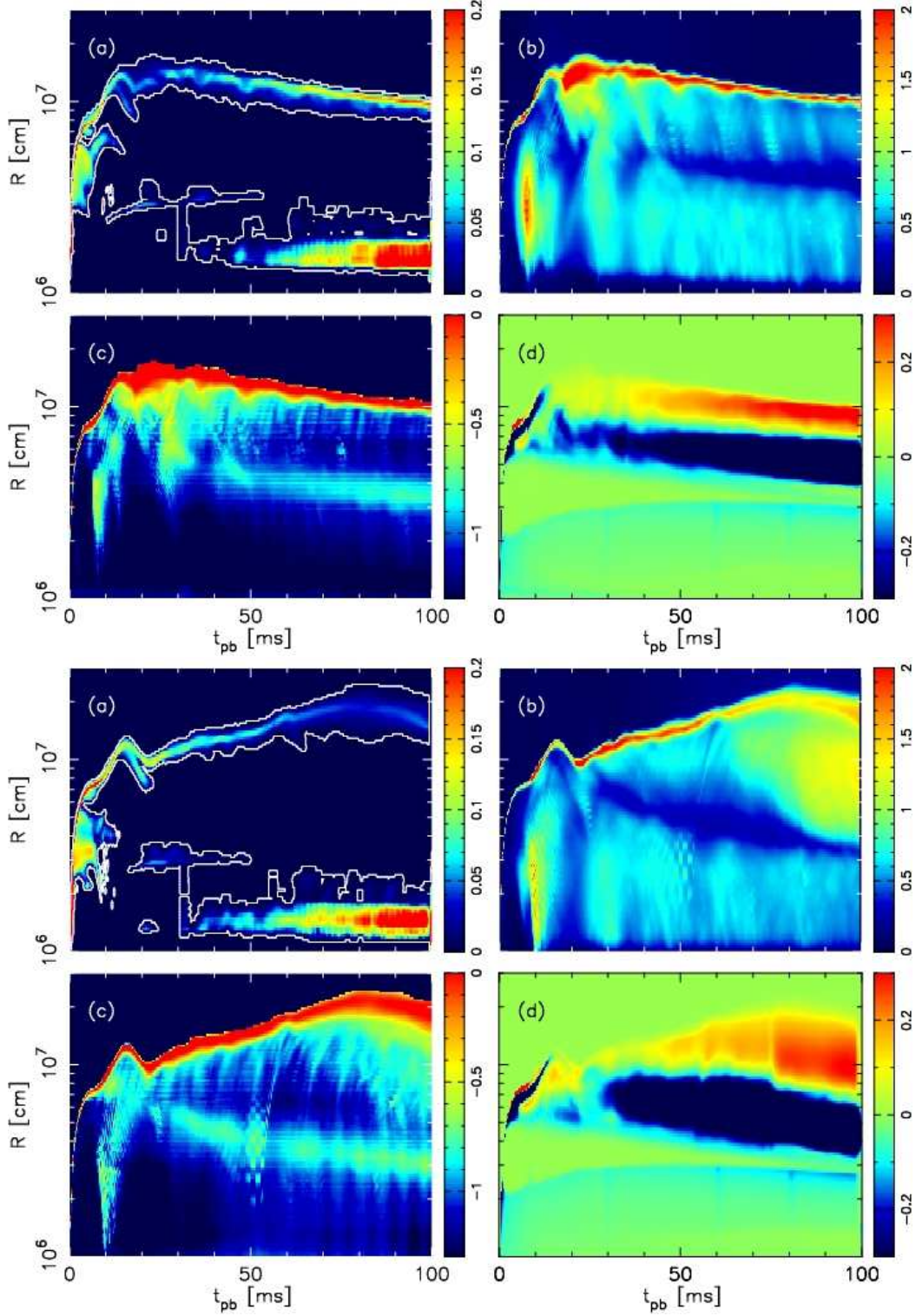


Fig. 9.— Panels (a) to (d) display angle averaged; (a) Brunt-Väisälä frequency (ω_{BV} ms^{-1}); (b) anisotropic velocity V_{aniso} normalized by 10^9 cm s^{-1} ; (c) normalized pressure perturbation Δp (in a logarithmic scale) (d) net energy deposition rate per baryon Q_{net} [$\text{GeV nuc}^{-1} \text{s}^{-1}$] for our 3D-SR (top four panels) and 3D-GR model (the rest four), respectively. Note that convectively unstable regions (i.e., $\omega_{\text{BV}} > 0$) are only shown in panel (a) and the white line represents the contour of $\omega_{\text{BV}} = 0$.

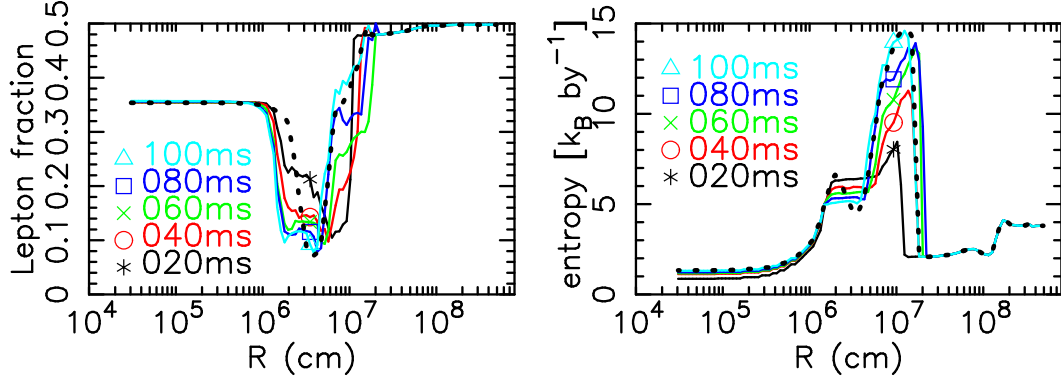


Fig. 10.— Profiles of angle-averaged total lepton fraction (*left*) and entropy (*right*) at times for the 3D-GR model, when the postbounce time is as indicated in the plots. Note that the result of 1D-GR model at $t_{\text{pb}} = 60$ ms (black dotted curve) is shown for comparison.

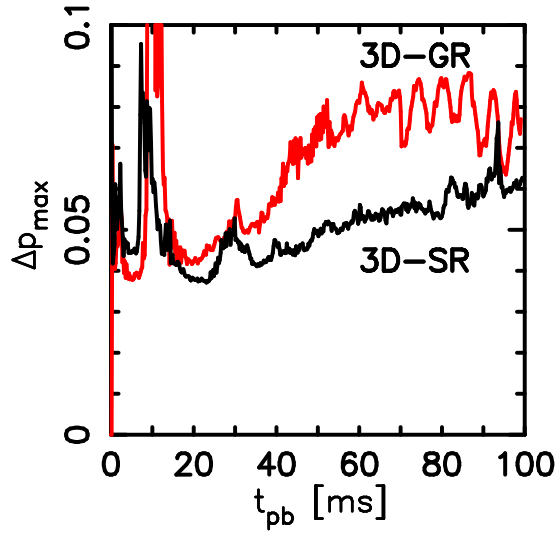


Fig. 11.— Evolution of the maximum of the pressure perturbation Δp_{max} in our 3D-SR and 3D-GR models. In taking the maximum, we set the radial range as $20 \leq R \leq 50$ km to cover the coupling radius (see panels (c) in Figure 9).

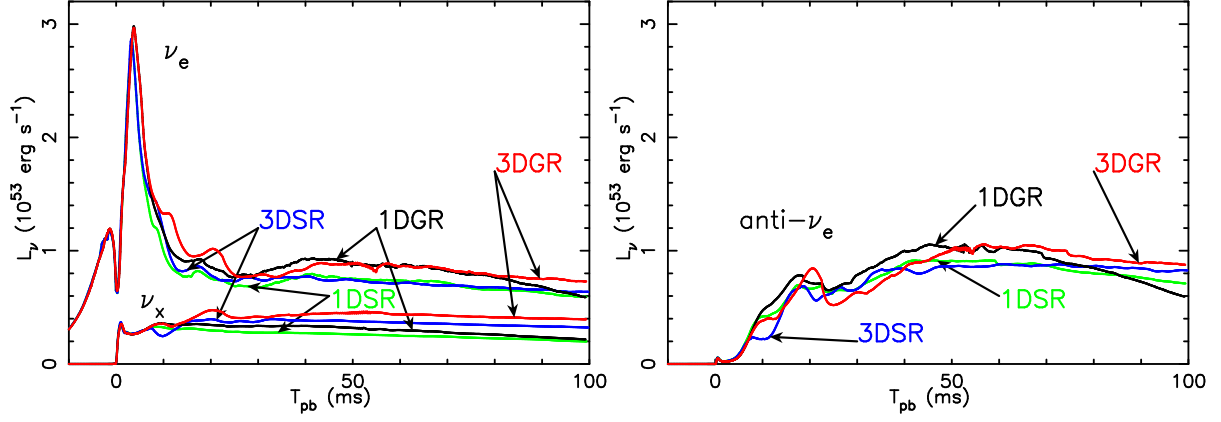


Fig. 12.— Neutrino luminosities of all neutrino flavors as a function of postbounce time (for ν_e , ν_x (left panel), and for $\bar{\nu}_e$ (right panel), respectively).

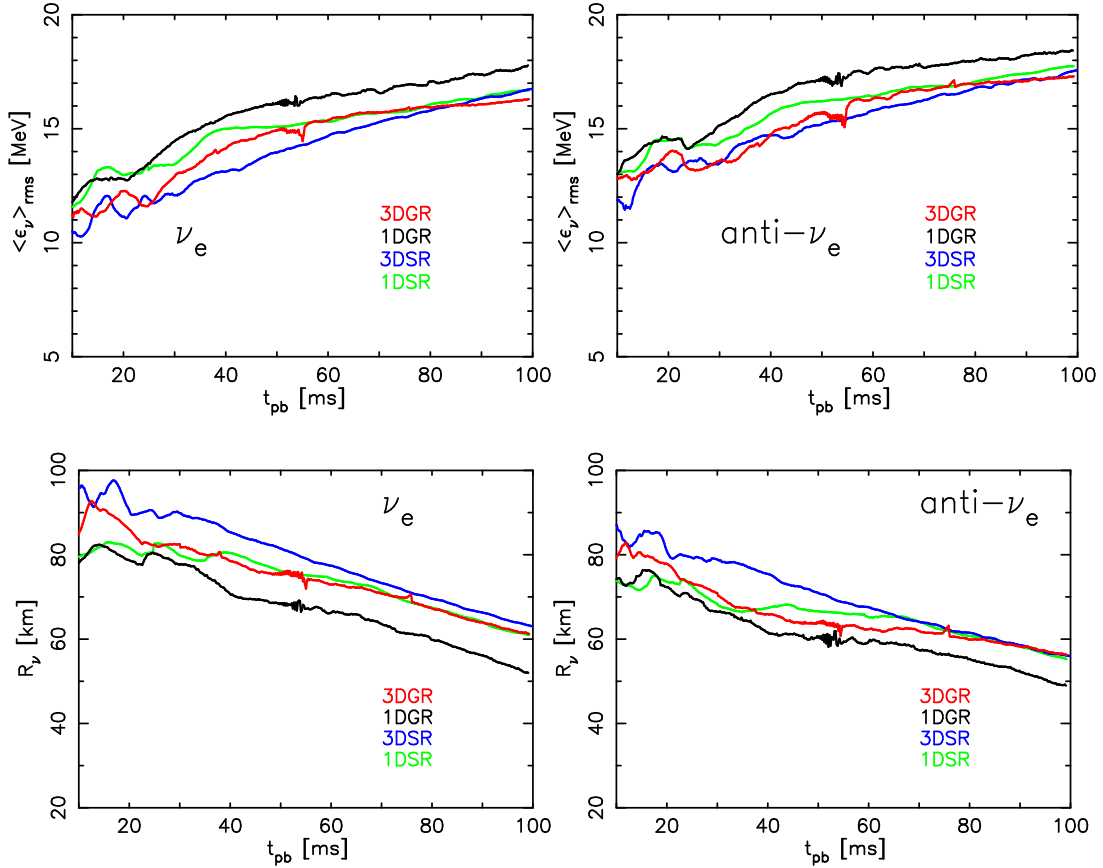


Fig. 13.— Evolution of the angle average RMS neutrino energy (*upper panels*) and the radii of the neutrino spheres (*lower panels*) for ν_e (left) and $\bar{\nu}_e$ (right). Colors are as in Figure 12.

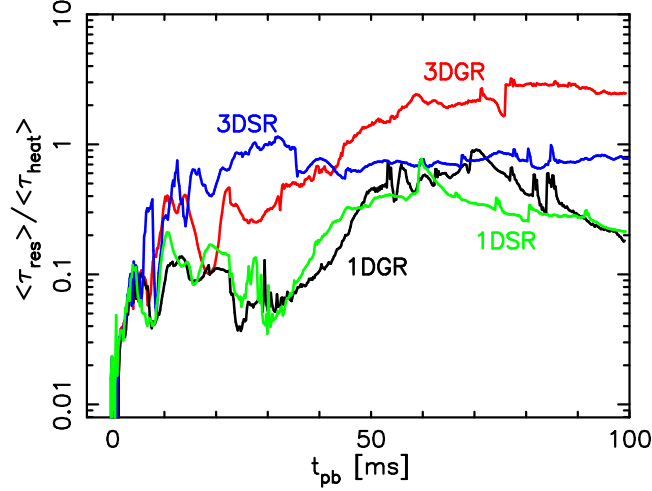


Fig. 14.— The ratio of the residency timescale to the heating timescale for the set of our models as functions of post-bounce time (see text for the definition of the timescales).

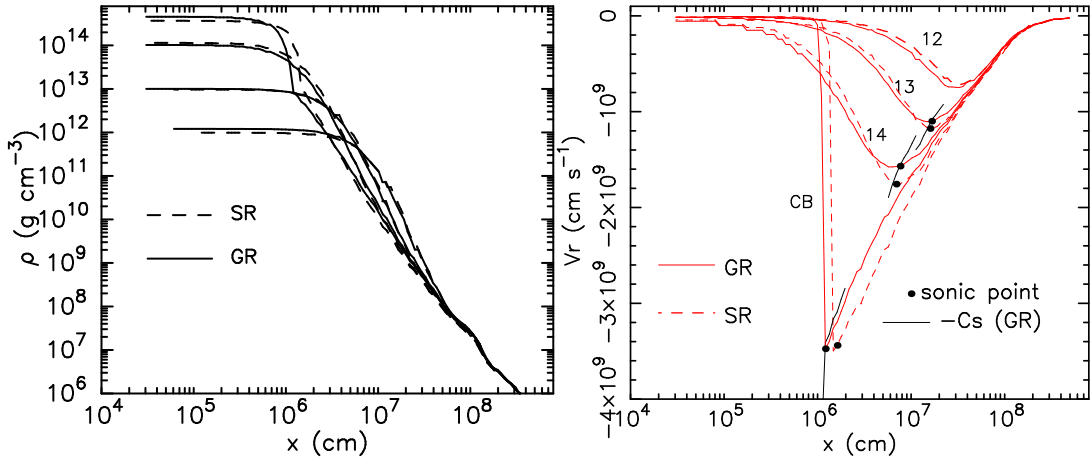


Fig. 15.— Profiles of the rest-mass density (*left*) and the radial velocity (*right*) at times, when the central density reaches at $10^{12,13,14}$ g cm $^{-3}$ (from the bottom up to the top in the left panel, each density is denoted by 12, 13, 14 and by “CB” at bounce in the right panel). Solid and dashed line is for the GR and SR model, respectively. In the right panel, the profiles of the sound velocity and the sonic point are indicated by black solid lines and black points at the intersection between the radial velocity and the sound velocity.

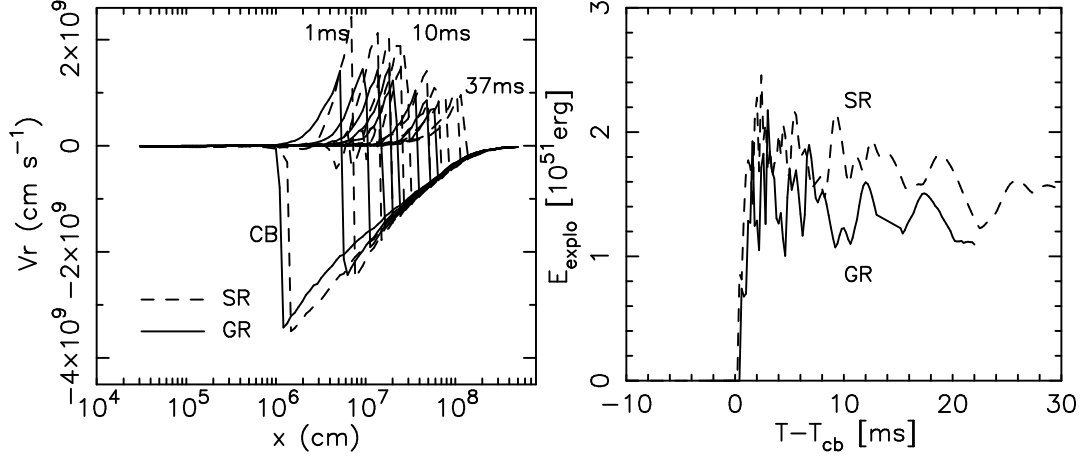


Fig. 16.— Evolution of the radial velocity (left panel) and the explosion energy (right panel) for the GR (solid line) and SR model (dashed line), respectively. “CB” (core bounce) and the postbounce time is shown for reference. Note that the explosion energy is defined in the Newtonian limit that refers to the integral of the energy over all zones that have a positive sum of the specific internal, kinetic and gravitational energy. A smaller inner-core mass in the GR model leads to a smaller explosion energy because the amount of dissociation of iron nuclei becomes larger during the shock propagation.

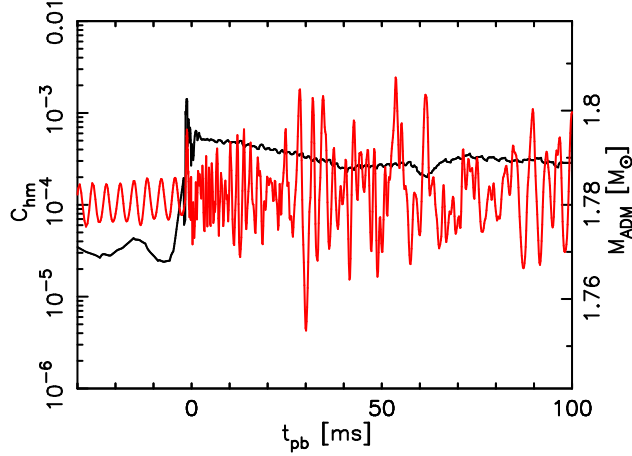


Fig. 17.— The violation of the Hamiltonian constraint C_{hm} (black) and the ADM mass (red) in our 3D-GR model are plotted against the postbounce time. Refining procedures and enforcing the Hamiltonian constraint are done every time the number of the AMR blocks is increased by the AMR procedure (see section 3) and every time a restart of the simulation is needed such as after the maintenance of the supercomputing system.

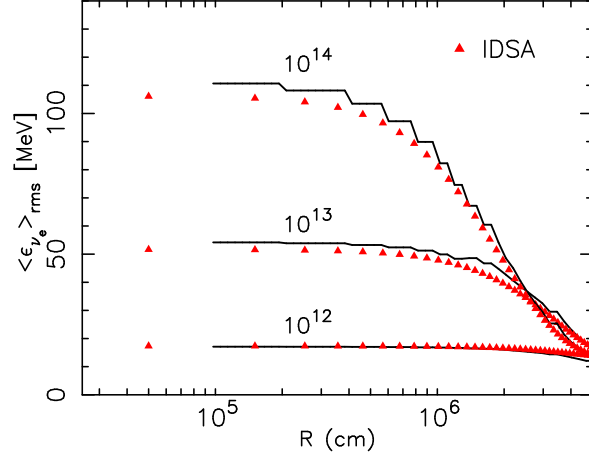


Fig. 18.— Profiles of the RMS neutrino energy at times, when the central density reaches $10^{12,13,14}$ g cm $^{-3}$ obtained by the present scheme (black solid line) and by the IDSA scheme (red triangle), respectively.

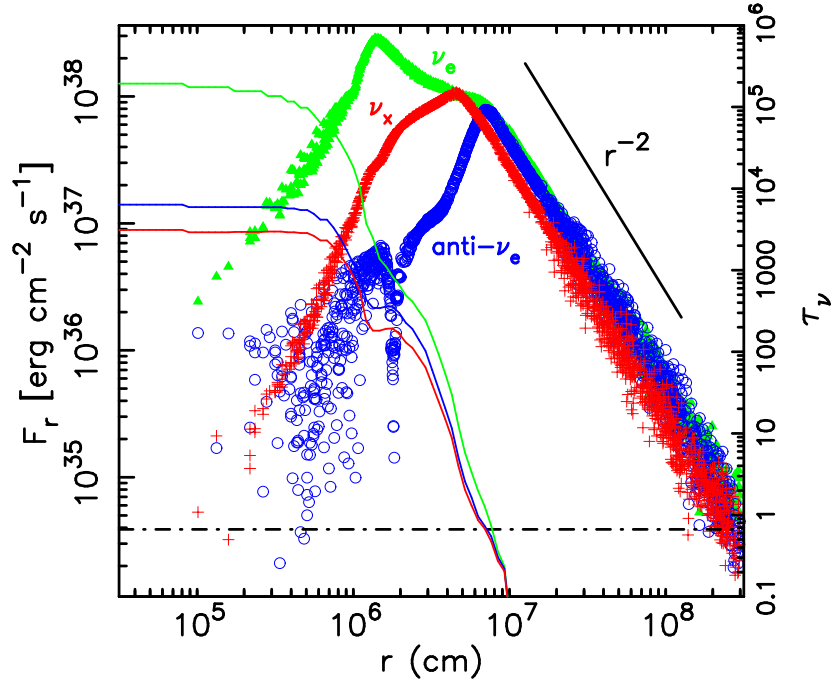


Fig. 19.— Radial components of neutrino energy flux $F_{r,\nu}$ in all flavors are plotted by color coded points (*green* ν_e , *blue* $\bar{\nu}_e$ and *red* ν_x). The profiles of optical depth τ_ν are shown by color-coded solid lines. The horizontal dash-dotted line is drawn for $\tau = 2/3$. As a reference, black solid line represents the slope of r^{-2} in the log-scale. The data are at 20ms after bounce in the 3D-GR model (see, section 3).

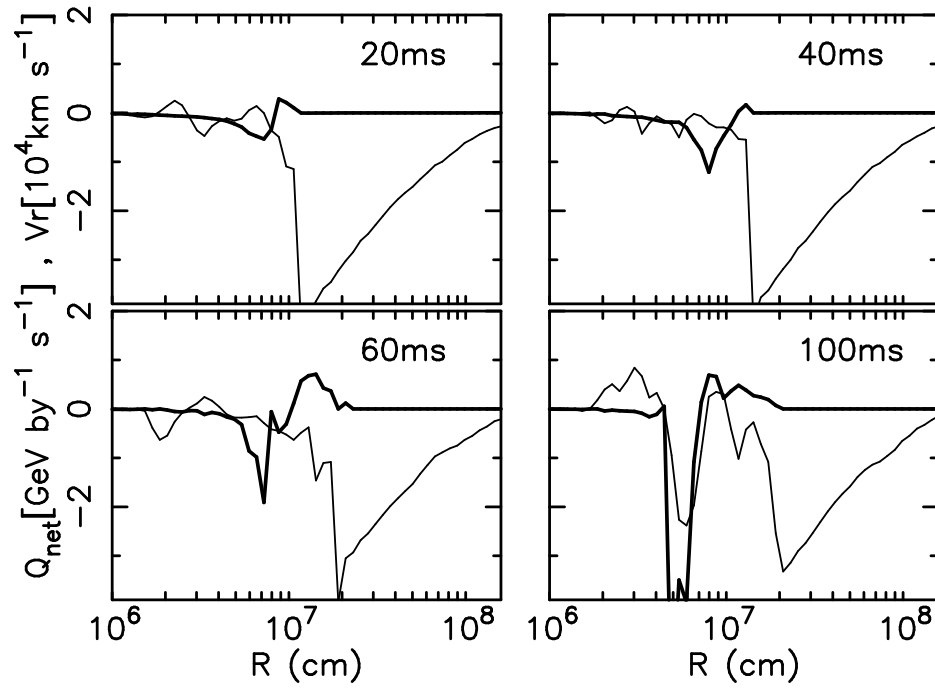


Fig. 20.— Profiles of the net heating rate $Q_{\text{net}} [\text{GeV by}^{-1} \text{s}^{-1}]$ (*thick*) and the radial velocity v_r (*thin*) along the x axis for our 3D-GR model at selected postbounce epochs. Note that v_r is normalized by 10^4 km s^{-1} .